# CH: ~~23~~ 20 Electric Field

charge ($q$): الشحنة الكهربائية (C)

$$q_{\text{الجسم}} = ne \begin{cases} 1.6 \times 10^{-19}\,C \Leftarrow \text{الجسم يفقد } e \text{ (lose)} \\ -1.6 \times 10^{-19}\,C \Leftarrow \text{الجسم يكتسب } e \text{ (accept)} \end{cases}$$

$n$: عدد الـ e المكتسبة أو المفقودة وهي عدد صحيح موجب

**Ex**: If an object accepts $2 \times 10^{7}\,e$, what is its charge?

$$q = ne$$
$$= 2 \times 10^{7} \times -1.6 \times 10^{-19}$$
$$= -3.2 \times 10^{-12}\,C.$$

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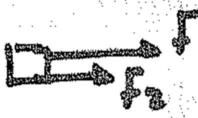
**Ex**: If an object Contains $4 \times 10^{3}\,e^-$ and $2 \times 10^{3}\,p^+$ what is its net charge?

$$q_{\text{الجسم}} = ne + np$$
$$= 4 \times 10^{3} \times -1.6 \times 10^{-19} + 2 \times 10^{3} \times 1.6 \times 10^{-19}$$
$$= -6.4 \times 10^{-16} + 3.2 \times 10^{-16}$$
$$= -3.2 \times 10^{-16}\,C.$$

٤) حساب محصلة المتجهات :-

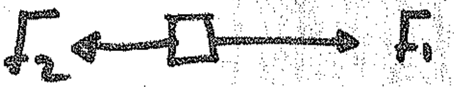
١) اذا كان المتجهين بنفس الاتجاه :

$F_{net} = F_1 + F_2$




٢) اذا كانت الكميتان المتجهتان متعاكستان :

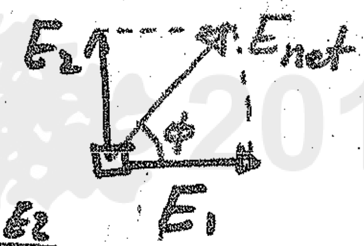
$\Rightarrow \boxed{F_{net} = F_1 - F_2}$ ← باتجاه الأكبر




٣) اذا كان المتجهان متعامدان :

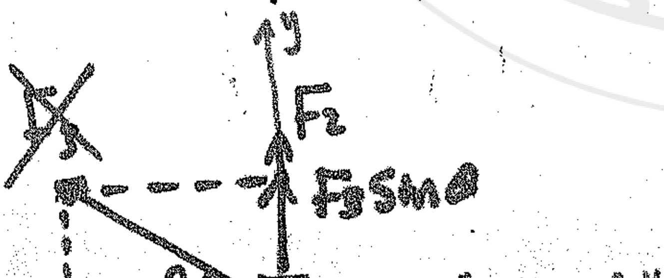
$E_{net} = \sqrt{E_1^2 + E_2^2}$

$\tan\phi = \frac{E_2}{E_1} \Rightarrow \phi = \tan^{-1}\frac{E_2}{E_1}$



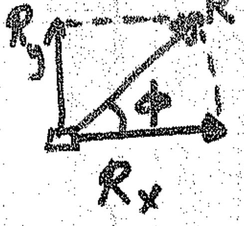

٤) اذا أثر على الجسم أكثر من متجهين ، أو اذا كانت $\theta \neq 90°$

١) تحديد المحاور :
٢) تحليل أي قوة لا تنطبق على المحاور
٣) اخذ المحصلة السينية والصادية :



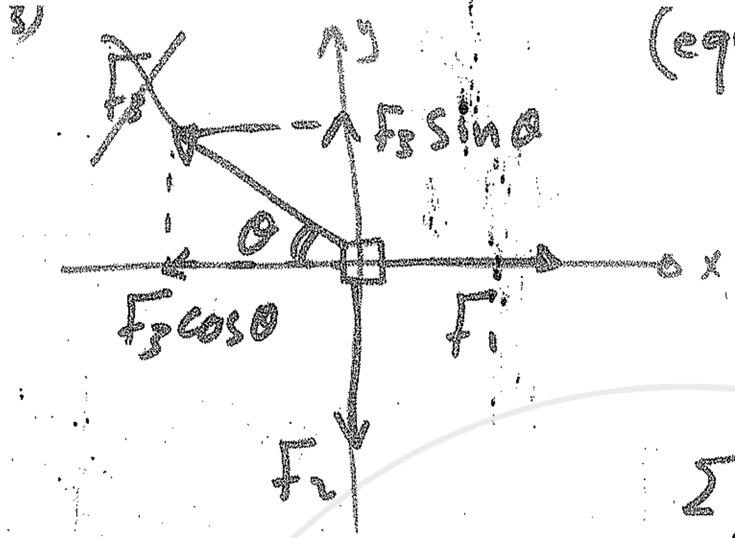

$R_x = F_1 - F_3 \cos\theta$

$R_y = F_2 + F_3 \cos\theta$




٤) أوجد R ، $\phi$ : $R = \sqrt{R_x^2 + R_y^2}$ ، $\tan\phi = \frac{R_y}{R_x}$

3)




ج) الجسم متزن ( equilibrium )

١) تحديد المحاور

٢) نحلل القوة الغير منطبقة على المحاور

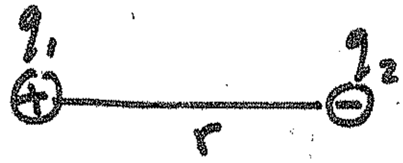
٣) نطبق قانوني الاتزان :-

$$\sum F_{\rightarrow} = \sum F_{\leftarrow} \quad , \quad \sum F_{\uparrow} = \sum F_{\downarrow}$$

$$F_1 = F_3 \cos\theta \qquad \Big\| \qquad F_3 \sin\theta = F_2$$

# Coulomb's law






$$F_e = \frac{k\, q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi \epsilon_0} = 9*10^9$$

$$\epsilon_0 = 8.85*10^{-12}$$

* الشحنة سالبة ، فإننا لا نعوض الإشارة السالبة في قانون كولوم

$K = 10^3$ , $M = 10^6$ , $G = 10^9$

$c = 10^{-2}$ , $m = 10^{-3}$ , $\mu = 10^{-6}$ , $n = 10^{-9}$ , $P = 10^{-12}$ , $f = 10^{-15}$

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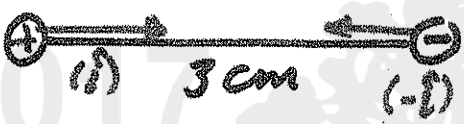
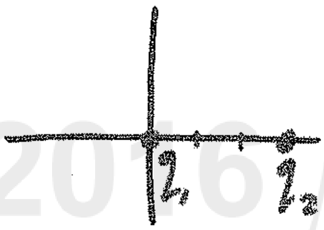
**Ex :** $q_1 = 8\mu C$ , $q_2 = -6\mu C$




$$F = \frac{9*10^9 * 8*10^{-6} * 6*10^{-6}}{(3*10^{-2})^2} = 48*10 = 480\,N \text{ (attraction)}$$

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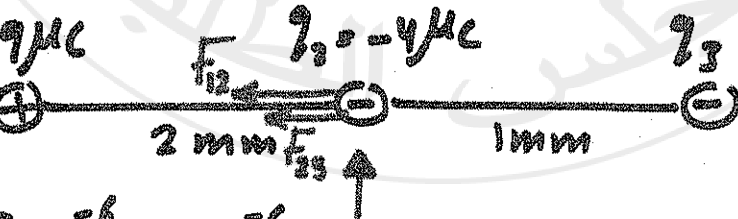
**Ex :** $q_1 = 9\mu C$ , $q_2 = -4\mu C$ , $q_3 = -1\mu C$

What is the net force that acting on $q_2$.



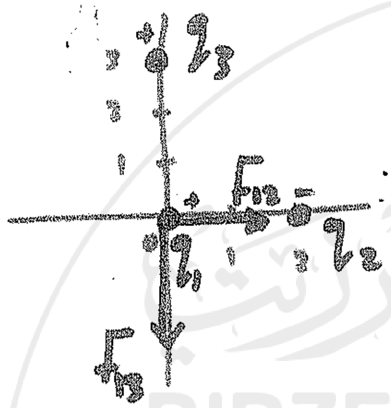

$$F_{12} = \frac{9*10^9 * 9*10^{-6} * 4*10^{-6}}{(2*10^{-3})^2} = 81*10^3\,N$$

$$F_{23} = \frac{9*10^9 * 4*10^{-6} * 1*10^{-6}}{(1*10^{-3})^2} = 36*10^3\,N$$

$$F_{net} = F_{12} + F_{23} = 117*10^3\,N\,(-\hat{i})$$

* اتجاهات
* حسابات
* محصلة.

**Ex**: If we have 3-charges, $q_1 = 1nC$ at origin and $q_2 = -4nC$ at $x = 2cm$, while $q_3 = 12nC$ at $y = 3cm$, Find the force acting on $\underline{q_1}$.




$$F_{12} = \frac{9\times10^{9} \times 1\times10^{-9} \times 4\times10^{-9}}{(2\times10^{-2})^2}$$

$$= 9\times10^{-5} N$$

$$F_{13} = \frac{9\times10^{9} \times 1\times10^{-9} \times 12\times10^{-9}}{(3\times10^{-2})^2}$$

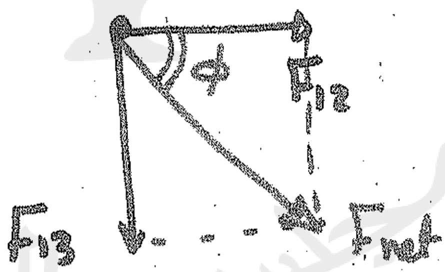
$$= 12\times10^{-5} N.$$

$$F_{net} = \sqrt{F_{12}^2 + F_{13}^2}$$

$$= 10^{-5}\sqrt{9^2 + 12^2}$$

$$= 10^{-5}\sqrt{81+144}$$

$$= 15\times10^{-5} N$$

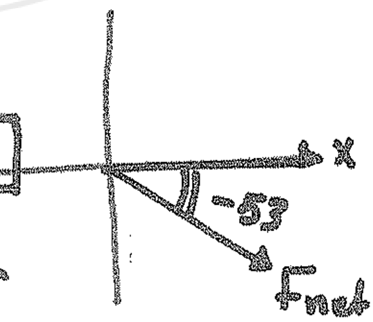



$$\tan\phi = \frac{12\times\cancel{10^{-5}}}{9\times\cancel{10^{-5}}}$$

$$= \frac{4}{3}$$

$$\Rightarrow \boxed{\phi = 53.1^\circ}$$

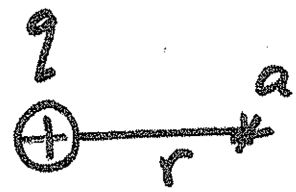
direction of $F_{net}$ is $\boxed{-53^\circ}$ or $307^\circ$.



# Electric Field



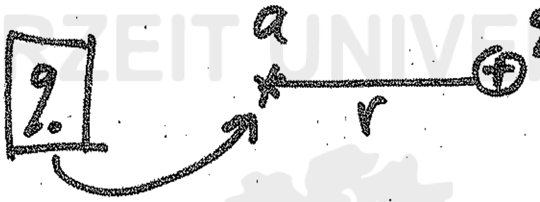
$$\boxed{E = \frac{k\,q}{r^2}}$$



* الإشارة السالبة لا تعوض.

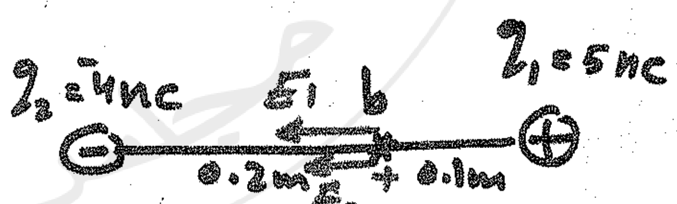
E مع اتجاه F
$q_0 = \oplus$

E عكس اتجاه F
$q_0 = \ominus$



$$\Rightarrow F = q_0 E_a$$

Ex: calculate the $\underline{E}$ at $\underline{b}$.



$$E_1 = \frac{9\times10^{9} * 5*10^{-9}}{(1*10^{-1})^2} = 45*10^{2}\ N/C$$

$$E_2 = \frac{9\times10^{9} * 4*10^{-9}}{(2*10^{-1})^2} = 9*10^{2}\ N/C$$

* اتجاهات.
* حسابات.
* محصلة.

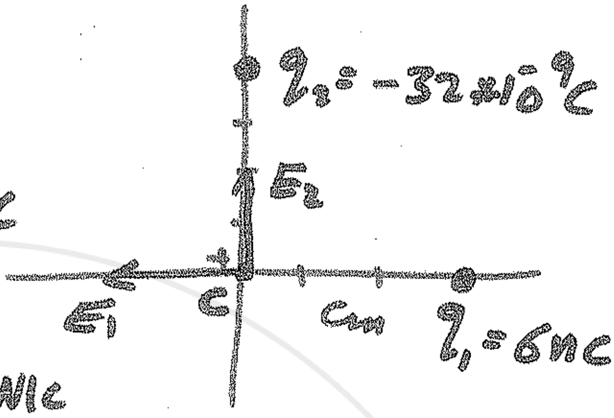
$$E_{net} = E_b = E_1 + E_2 = 54*10^{2}\ N/C\ (-\hat{i})$$

$$F = q_0 E_b = 2*10^{-3} * 45*10^{2} = 90*10^{-1} = 9N$$

b ⟵ $q_0$ *

**Ex**: Calculate the Electric Field at C.

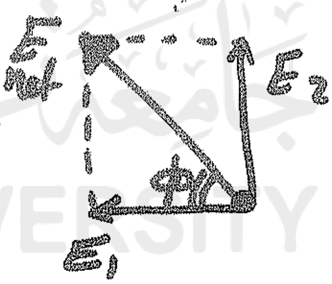
**Sol**:




$$E_1 = \frac{9\times10^{9}\times6\times10^{-9}}{(3\times10^{-2})^2} = 6\times10^{4}\ N/C$$

$$E_2 = \frac{9\times10^{9}\times32\times10^{-9}}{(4\times10^{-2})^2} = 18\times10^{4}\ N/C$$

$$E_{net} = \sqrt{E_1^2 + E_2^2}$$
$$= \sqrt{6^2+18^2}\times10^{4}$$
$$= 19\times10^{4}\ N/C$$




$$\tan\phi = \frac{18\times10^{4}}{6\times10^{4}} = \cancel{1.5}\ 3 \Rightarrow \boxed{\phi = 71.5^\circ}$$

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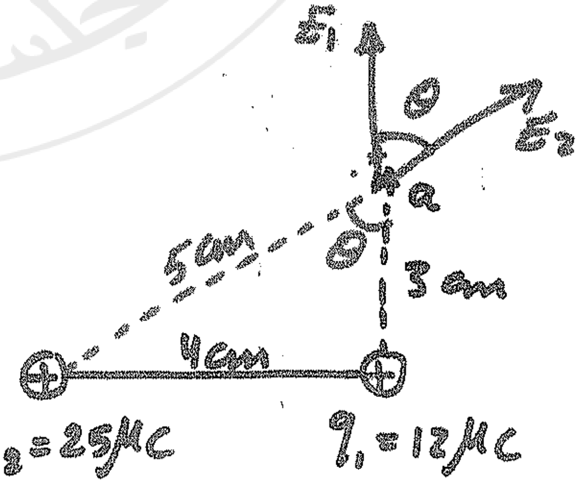
**Ex**: What is the Electric field at Point a:

$$E_1 = \frac{9\times10^{9}\times12\times10^{-6}}{9\times10^{-4}} = 12\times10^{7}\ N/C$$

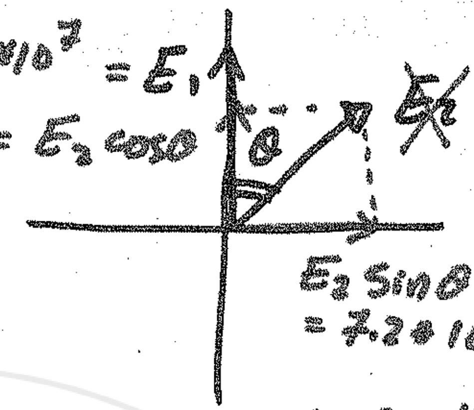
$$E_2 = \frac{9\times10^{9}\times25\times10^{-6}}{(5\times10^{-2})^2} = 9\times10^{7}\ N/C$$



$R_x = 7.2 \times 10^7 \ (+\hat{i})$

$R_y = 12 \times 10^7 + 5.4 \times 10^7$

$= 17.4 \times 10^7 \ (+\hat{j})$

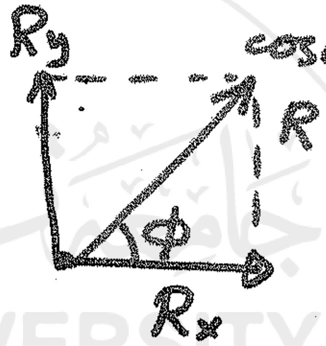



$\sin\theta = \frac{4}{5} = 0.8$

$\cos\theta = \frac{3}{5} = 0.6$

$R = \sqrt{7.2^2 + 17.4^2} \times 10^7$

$= 18.8 \times 10^7 \ N/C$




$\tan\phi = \frac{17.4 \times 10^7}{7.2 \times 10^7} = 2.4$

$\Rightarrow \phi \simeq 67.4$

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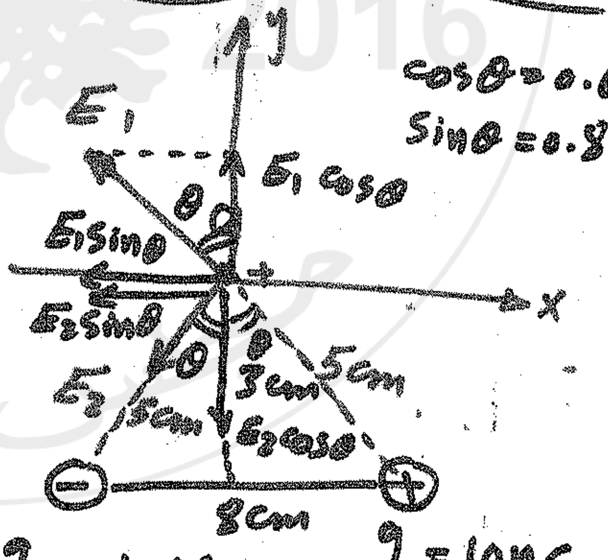
**Ex**: Find $E_{net}$ at $\underline{a}$:

$\cos\theta = 0.6$

$\sin\theta = 0.8$




$E_1 = \frac{9 \times 10^9 \times 10 \times 10^{-9}}{25 \times 10^{-4}} = 3.6 \times 10^4 \ N/C$

$E_2 = \frac{9 \times 10^9 \times 10 \times 10^{-9}}{25 \times 10^{-4}} = 3.6 \times 10^4 \ N/C$

$q_2 = -10nC \qquad q_1 = 10nC$

$R_x = E_1 \sin\theta + E_2 \sin\theta$

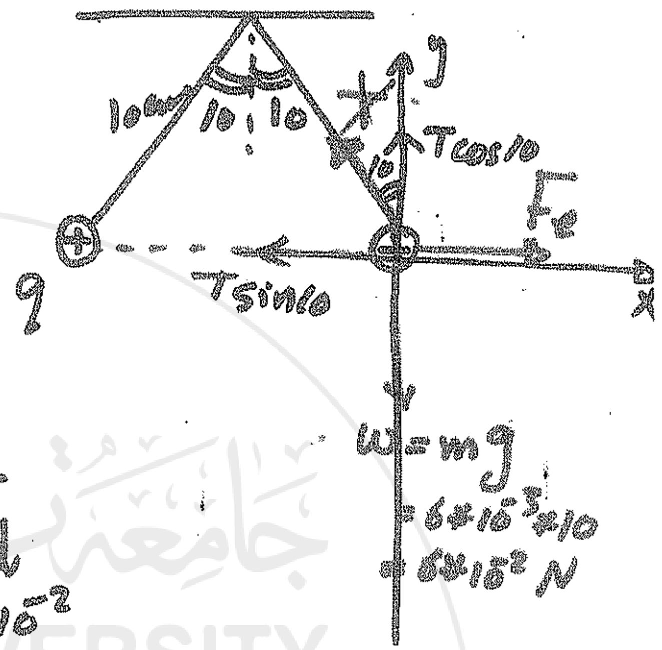
$= 3.6 \times 10^4 \times 0.8 + 3.6 \times 10^4 \times 0.8 = 2.88 \times 10^4 \ N/C \ (-\hat{i})$

$R_y = E_1 \cos\theta - E_2 \cos\theta = 0$

$\Rightarrow R = R_x = 2.88 \times 10^4 \ N/C \ \boxed{-\hat{i}}$

Ex Find $q$ if the system is at equilibrium. ⑨


$\Longrightarrow$




$$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$$

$$T\cos 10 = 6 * 10^{-2}$$

$$\boxed{T = 0.0609 \text{ New}}$$

$$\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$$

$$F_e = T \sin 10$$

$$= 0.0609 * \sin 10$$

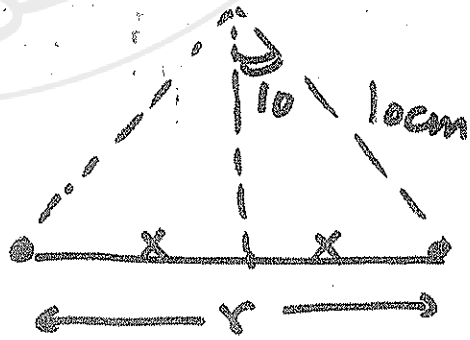
$$\boxed{F_e = 1.06 * 10^{-2} \text{ New}}$$

$$F_e = \frac{9*10^9 \, q_1 q_2}{r^2}$$

$$1.06*10^{-2} = \frac{9 \times 10^9 * q^2}{(3.4*10^{-2})^2}$$

$$q^2 = 1.36 * 10^{-7}$$

$$\boxed{q = 3.7 * 10^{-4} \, C}$$




$r = 2x$ $\quad \sin 10 = \frac{x}{10*10^{-2}}$

$\Rightarrow x = 1.7*10^{-2} \, m$

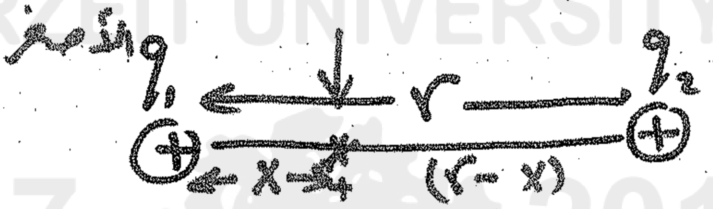
$\Rightarrow r = 2x = 3.4*10^{-2} \, m$

# equilibrium Point

* هي النقطة التي ينعدم عندها المجال الكهربائي والقوة الكهربائية . ( $E = 0$ )

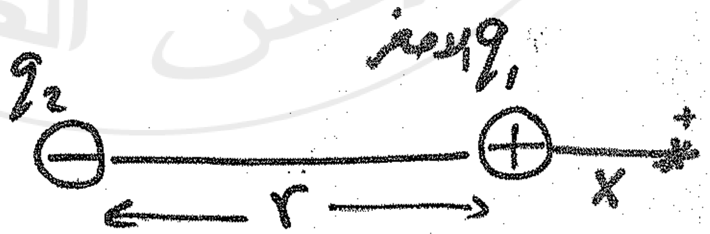
- شحنتان متشابهتان : نقطة التعادل تقع بينهما وأقرب للشحنة الأصغر .

- شحنتان مختلفتان : نقطة التعادل تقع خارجهما أقرب للأصغر




$\boxed{E_1 = E_2}$ ⟸ عندما تكون نقطة التعادل

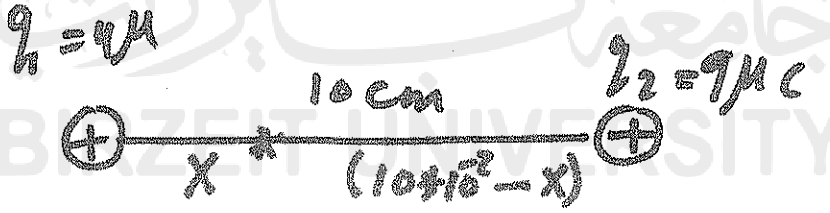
$$\frac{k q_1}{r_1^2} = \frac{k q_2}{r_2^2}$$



**Ex**: If we two Point charges:

$q_1 = 4\mu C$ , $q_2 = 9\mu C$ , $r = 10\,cm$

where is the equilibrium Point.
(where is the Point that has <u>no</u> net electric field)




$$E_1 = E_2$$

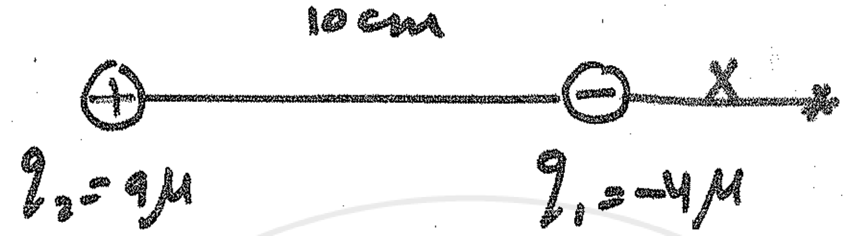
$$\frac{\cancel{K}\, 4*\cancel{10^{-6}}}{X^2} = \frac{\cancel{K}\, 9*\cancel{10^{-6}}}{(10*10^{-2} - X)^2}$$

$$\frac{2}{X} = \frac{3}{10*10^{-2} - X} \Longrightarrow 3X = 20*10^{-2} - 2X$$

$$5X = 20*10^{-2}$$

$$\boxed{X = 4*10^{-2}\,m = 4\,cm}$$

**Ex:** $q_1 = -4\mu C$ , $q_2 = 9\mu C$ , $r = 10\,cm$




$$E_1 = E_2$$

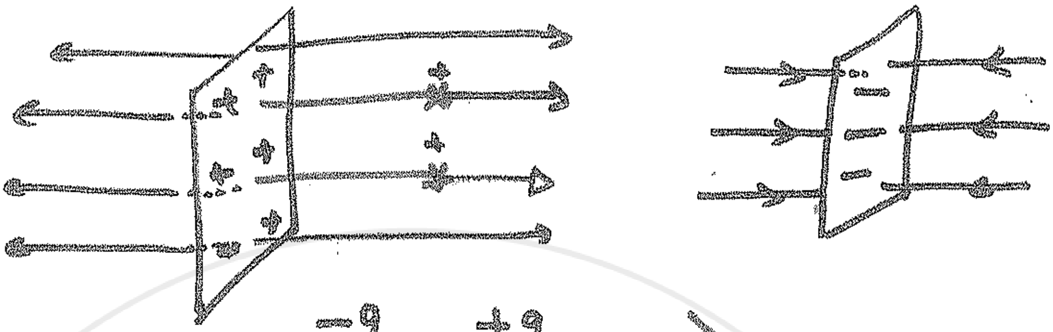
الجذر

$$\frac{\cancel{9\times10^{9}}\times 4\times\cancel{10^{-6}}}{X^2} = \frac{\cancel{9\times10^{9}}\times 9\times\cancel{10^{-6}}}{(10\times10^{-2}+X)^2}$$

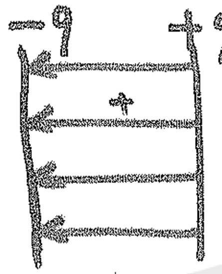
$$\frac{2}{X} = \frac{3}{10\times10^{-2}+X} \Longrightarrow 3X = 20\times10^{-2} + 2X$$

$$\boxed{X = 20\times10^{-2}\,m = 20\,Cm}$$

# Uniform Electric Field




$E$ = Constant




$\cancel{E_{Uni} = \frac{9\times10^{9} * q}{r^2}}$

تطبيق $\boxed{1}$

حركة جسيم مشحون داخل هذا المجال:

Motion of a small charged Particle inside the uniform electric Field.


$$F_E = qE = ma$$

$$W = F X \cos\theta_{FX}$$

$$\boxed{\Delta K = K_f - K_i = \tfrac{1}{2}mV_f^2 - \tfrac{1}{2}mV_i^2}$$

$$W_{total} = \Delta K$$

$$V_2 = V_1 + at$$

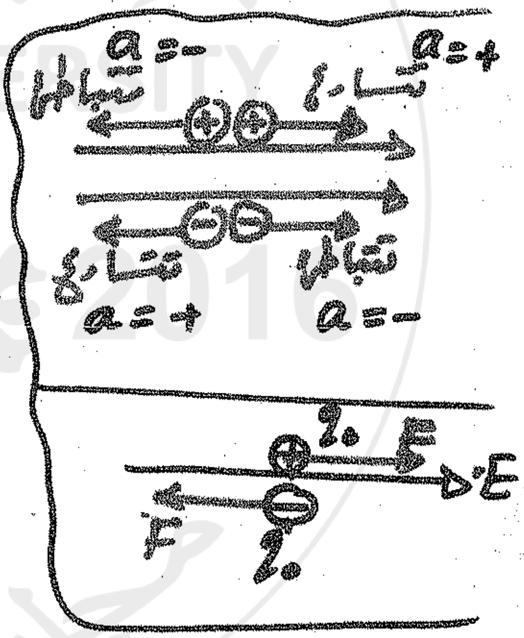
$$V_2^2 = V_1^2 + 2aX$$

$$X = V_1 t + \tfrac{1}{2}at^2$$

$$X = \left(\frac{V_1+V_2}{2}\right)t$$

**Ex**: An object of mass $\underset{m}{\underline{2*10^{-4}\,\text{kgm}}}$ and charge of $\underset{q_0}{\underline{6*10^{-8}\,C}}$, enters ~~as~~ a Uniform electric field with speed of $\underset{V_1}{\underline{2*10^{4}\,m/s}}$ for $\underset{t}{\underline{10^{3}\,sec}}$.
if $\underset{E}{\underline{E = 4000}}$ N/C as in figure, Find:

1) acceleration
2) Final speed
3) traveled displacement.
4) Force ~~done~~ exerted.
5) Work done.
6) change in Kinetic energy.


**Sol**:

1) $q_0 E = ma$

$6*10^{-8} * 4*10^{3} = 2*10^{-4} a$

$a = 12 * 10^{-1} = 1.2\ m/s^2$.

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2) $V_2 = V_1 + at$

$= 2*10^{4} + 1.2*10^{3}$

$= 20000 + 1200$

$\boxed{V_2 = 21200\ m/s.}$

3) $X = V_1 t + \frac{1}{2} a t^2$

$= 2*10^{4}*10^{3} + \frac{1}{2}*1.2*10^{6}$

$= 2*10^{7} + 0.6*10^{6}$

$\boxed{X = 20.6*10^{6}\ m}$

4) $F = q_o E = ma$

$= 6*10^{-8} * 4*10^{3}$

$= 24*10^{-5}\ N$

$= 2*10^{-4} * 1.2$

$= 2.4*10^{-4}$

$= 24*10^{-5}\ N$

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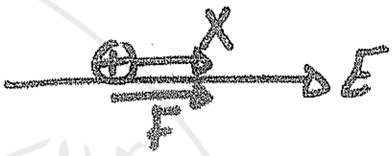
5) $W = F\ X \cos\theta_{FX}$




$= 24*10^{-5} * 20.6*10^{6} \cos 0$

$= 4944\ J$

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6) $\Delta K = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$

$= \frac{1}{\cancel{2}} \cancel{2}*10^{-4} * (21200)^2 - \frac{1}{\cancel{2}} * \cancel{2}*10^{-4} * (2*10^{4})^2$

$= 44944 - 40000$

$= 4944\ J$

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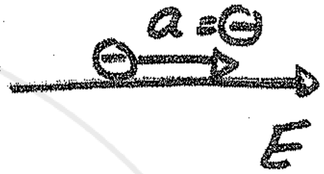
**Ex:** If an object of mass $\underset{m}{\underline{0.1\ gm}}$ and charge $\underset{q_o}{\underline{-2*10^{-6}\ C}}$, start moving with speed of $\underset{V_1}{\underline{20\ m/s}}$ inside a uniform electric field and with the field direction, the distance traveled is $\underset{X}{\underline{200\ m}}$ untill the particle was $\underset{V_2 = 0}{\underline{\text{stopped}}}$, what is the mag. of $\underset{E}{\underline{\text{Electric field}}}$.

Sol :

$$V_2^2 = V_1^2 + 2ax$$

$$0 = 400 + 2 * a * 200$$

$$a = -\frac{400}{400} = -1 \, m/s^2$$




$$q_0 E = ma$$

$$E = \frac{ma}{q_0} = \frac{0.1 * 10^{-3} * 1}{2 * 10^{-6}} = 0.5 * 10^{2}$$

$$= 50 \, N/C$$

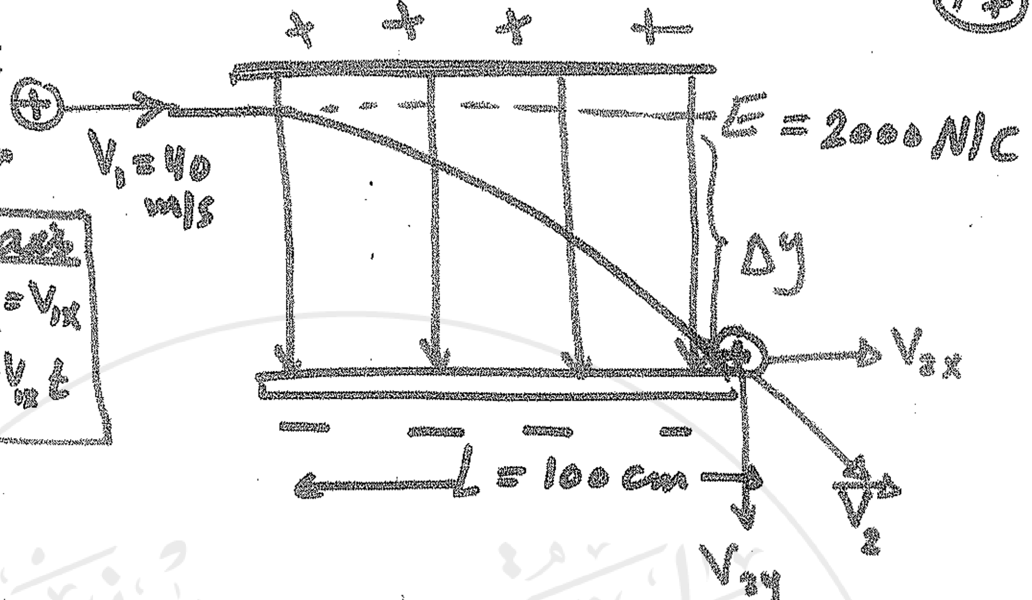
$$* \quad V_2 = V_1 + at$$

$$0 = 20 - 1t$$

$$\boxed{t = 20 \text{ sec}}$$

**Ex:** $q_0 = 10\,\mu C$

$m = 2\,gm$




| y-axis | x-axis |
|---|---|
| $V_{2y} = V_{1y} + a_y t$ | $V_{2x} = V_{1x}$ |
| $V_{2y}^2 = V_{1y}^2 + 2a_y y$ | $x = V_{1x} t$ |
| $y = V_{1y} t + \frac{1}{2} a_y t^2$ | |

Find:

1) acceleration ($a_y$)
2) final Velocity (speed).
3) Vertical displacement ($\Delta y$)
4) time.

---

$V_{1x} = V_1 \cos\theta = 40 \cos 0 = 40$ m/s

$V_{1y} = V_1 \sin\theta = 40 \sin 0 = 0$ m/s.

1) $q_0 E_y = m a_y$

$10 \times 10^{-6} \times 2000 = 2 \times 10^{-3} a_y$

$\boxed{a_y = {}^{+}10 \text{ m/s}^2}$

2) $V_{2x} = V_{1x} = 40$

$V_{2y} = \cancel{V_{1y}} + a_y t$

$= 10 \times 25 \times 10^{-3}$

$= 0.25$ m/s.

$x = V_{1x} t$

$1 = 40 t$

$t = 25 \times 10^{-3}$ sec

$\vec{V}_2 = 40\hat{i} - 0.25\hat{j}$

$S = |\vec{V}_2| = \sqrt{40^2 + 0.25^2} =$

3) $\Delta y = \cancel{V_{1y}t} + \frac{1}{2} g t^2$

$= \frac{1}{2} * 10 * (0.025)^2$

$= 3.125 * 10^{-3}\,m$

$= 3.125\ mm.$

---

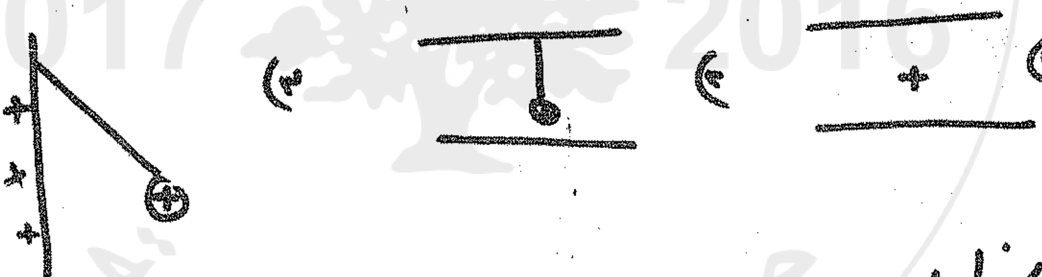
4) $t = 25 * 10^{-3}\ sec.$

$\Delta y = V_{1y}t + \frac{1}{2}at^2$

$\rightarrow \Delta x = V_{1x}\,t$

---

٣ تطبيق : اتزان الجسم المشحون داخل المجال المنتظم



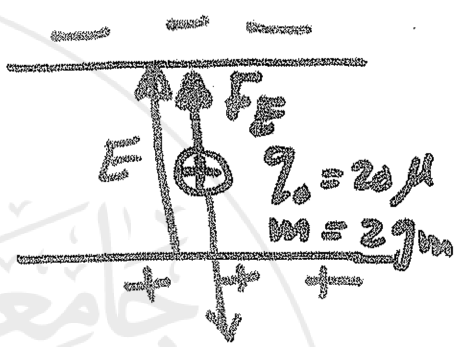
الخطوات :

١) نحدد اتجاه القوى المؤثرة .

٢) نحدد محاور مناسبة ونحلل القوة غير منطبقة على المحاور .

٣) نطبق قوانين الاتزان : $\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$ , $\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$

**Ex:** particle of mass 2gm and charge of 20μC, what is the magnitude and direction of $\underline{E}$ when $q_0$ is at equilibrium.




$$\sum F_{\uparrow} = \sum F_{\downarrow}$$

$$q_0 E = \omega$$

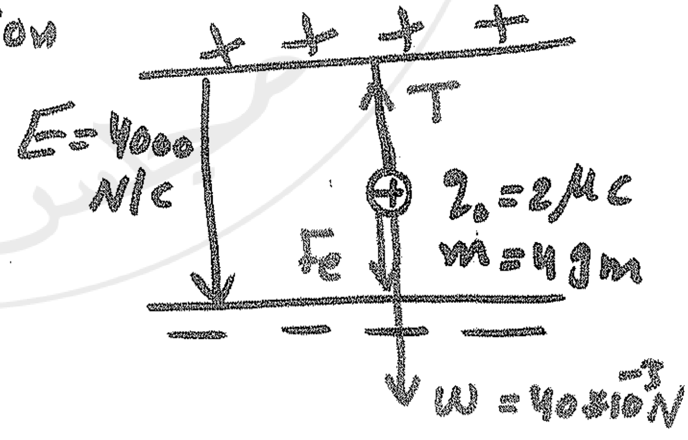
$$20 \times 10^{-6} E = 2 \times 10^{-2}$$

$$\boxed{E = 1 \times 10^{3}\ N/C}$$

$$\omega = mg$$
$$= 2 \times 10^{-3} \times 10$$
$$= 2 \times 10^{-2}\ N$$

---

**Ex:** what is the tension in the Cord.



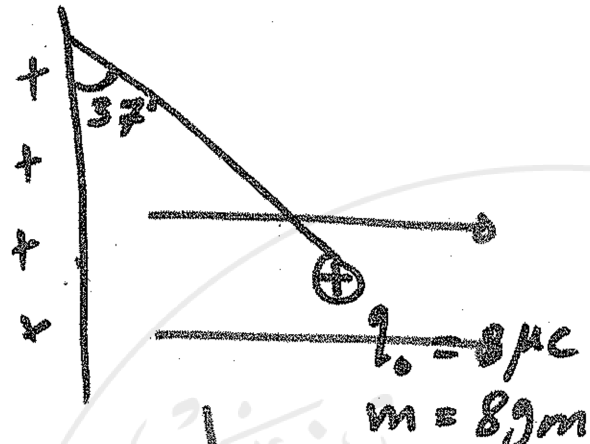

$$\sum F_{\uparrow} = \sum F_{\downarrow}$$

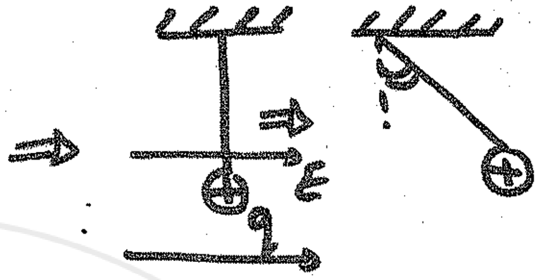
$$T = F_e + \omega$$

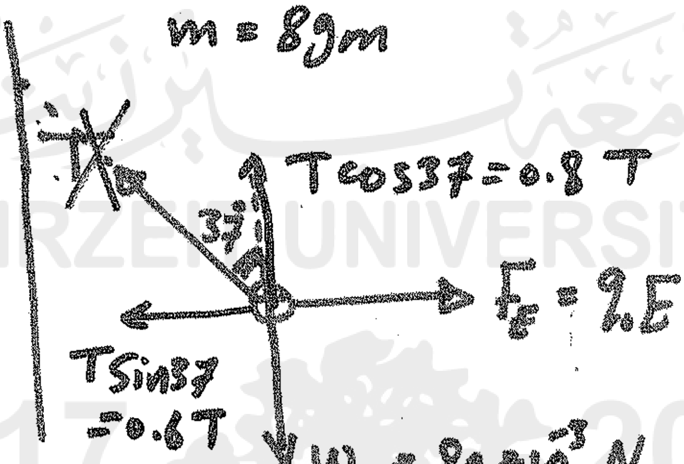
$$T = 48 \times 10^{-3} N.$$

$$F_e = q_0 E$$
$$= 2 \times 10^{-6} \times 4 \times 10^{3}$$
$$= 8 \times 10^{-3} N$$

**Ex:** Find $\underline{E}$




$$\sum F_{\uparrow} = \sum F_{\downarrow}$$

$$0.8\ T = 80 * 10^{-3}$$

$$\boxed{T = 0.1\ New}$$

$$\sum F_{\rightarrow} = \sum F_{\leftarrow}$$

$$q_0 E = T\ 0.6$$

$$E = \frac{0.6 * 0.1}{8 * 10^{-6}}$$

$$= \frac{6 * 10^{-2}}{8 * 10^{-6}} = 7.5 * 10^{3}\ N/C$$

لوال مش معطاه

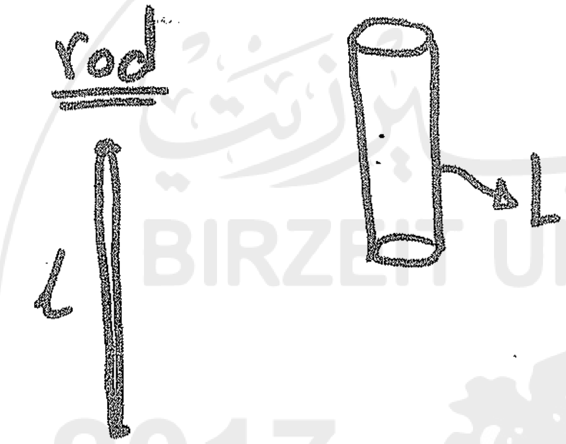
$$E = 400\ N/C\ (-\hat{i})$$

# Electric Field due to distribution of charge. 

المجال الناتج عن توزيع من الشحنات.

1) linear charge dist.

rod



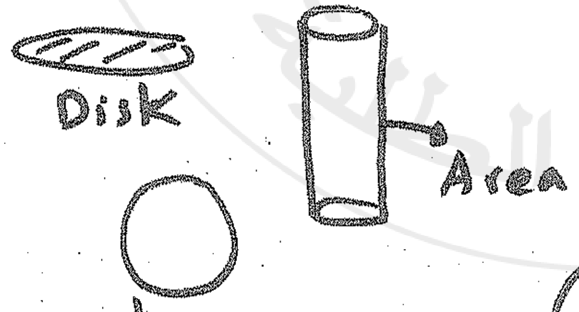
$L$ $\Rightarrow$ linear charge density ($\lambda$)

كثافة شحنة طولية.

$$\boxed{q = \lambda L}$$

$$dq = \lambda \, dL$$

2) Surface charge dist :-



Disk

sphere

Area $\Rightarrow$ Surface charge density

كثافة شحنة سطحية

($\sigma$)

$$dq = \sigma \, dA \Leftarrow \boxed{q = \sigma A}$$

3) Volume. charge dist.

$\Rightarrow$ Volume charge density

($\rho$) كثافة شحنة حجمية

$$dq = \rho \, dV \Leftarrow q = \rho \, Vol$$

$D = 2r$

D → diameter

r → radius

محيط الدائرة = $2\pi r$

مساحة الدائرة = $\pi r^2$

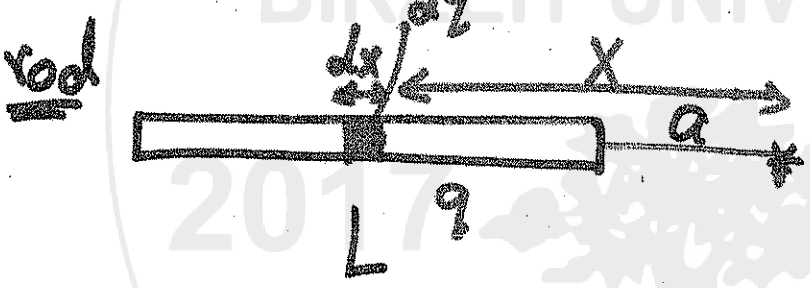
مساحة الكرة = $4\pi r^2$

حجم الكرة = $\frac{4}{3}\pi r^3$

المساحة الجانبية للاسطوانة = $2\pi r l$

حجم الاسطوانة = $\pi r^2 l$






$q = \lambda L$

$dq = \lambda\, dx$

$$dE = \frac{k\,dq}{r^2} = \frac{k \lambda\, dx}{x^2}$$

$$\boxed{\lambda = \lambda_0 x}$$

$$E = \int_{a}^{L+a} \frac{k\lambda\, dx}{x^2} = k\lambda \int_a^{L+a} \frac{dx}{x^2} = k\lambda \left[-\frac{1}{x}\right]_a^{L+a}$$

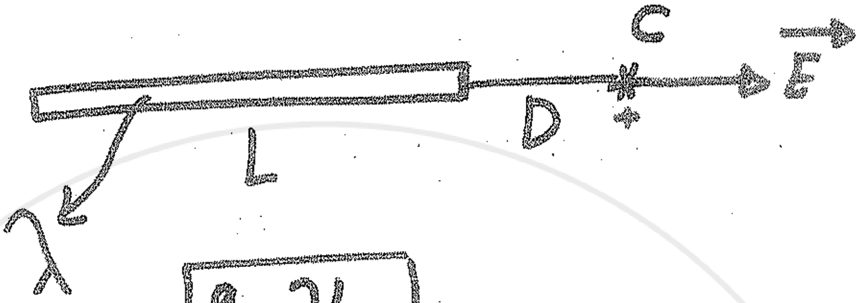
$$= k\lambda\left[\frac{-1}{L+a} - \frac{-1}{a}\right] = k\lambda\left[\frac{1}{a} - \frac{1}{L+a}\right]$$

$$= k\lambda\left[\frac{L+\cancel{a}-\cancel{a}}{a(L+a)}\right] = \frac{k\lambda L}{a(L+a)}$$

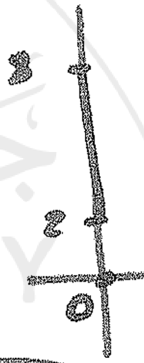
$\lambda L \rightarrow q$

# 1 rod:

a)




$$\boxed{q = \lambda L}$$

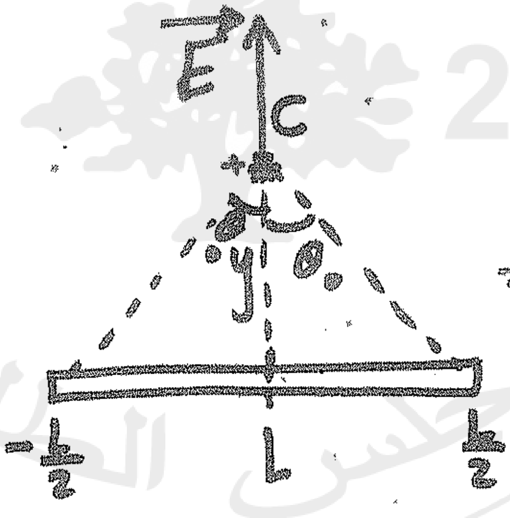



rod
$L = 6\,cm$
$q = 12\,\mu C$
$D = 2\,cm$

$$\Rightarrow E_c = \frac{k \lambda L \rightarrow q}{D(D+L)}$$

b)

finite rod



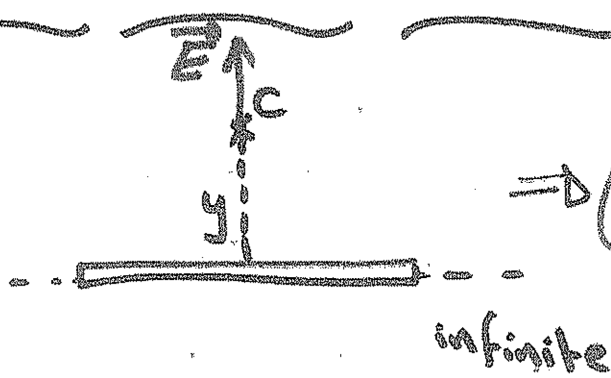

$$\Rightarrow E = \frac{2k\lambda \sin\theta_0}{y}$$

or

$$E = \frac{2k\lambda L}{2y\sqrt{y^2 + \frac{L^2}{4}}}$$

c)

infinite rod

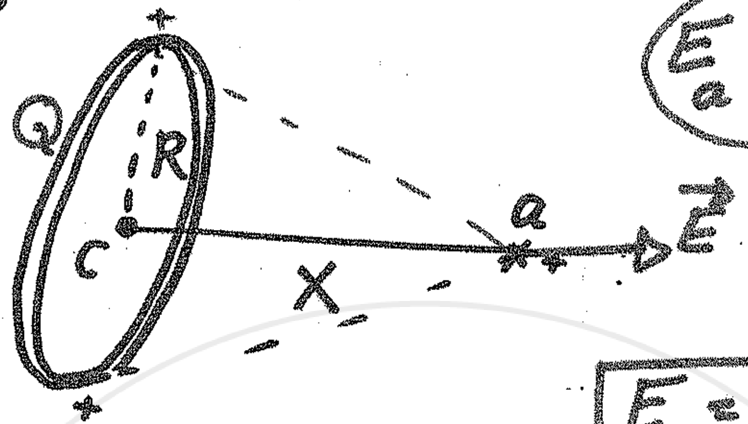



$$\Rightarrow E_c = \frac{2k\lambda}{y}$$

## [2] Ring

$\lambda \rightarrow Q = \lambda L$

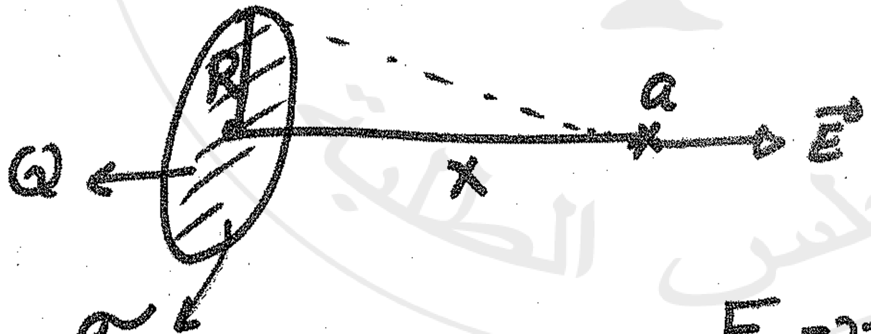
$Q = \lambda(2\pi R)$




$$E_a = \frac{kQX}{(X^2+R^2)^{3/2}}$$

$$\boxed{E_c = 0}$$


## [3] Disk




$Q = \sigma A$

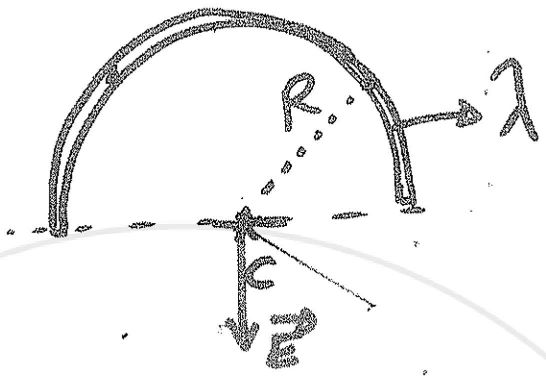
$= \sigma \pi R^2$

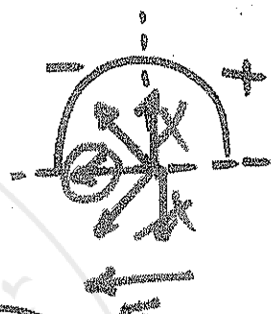
$$E = 2\pi k\sigma \left[1 - \frac{X}{\sqrt{R^2+X^2}}\right]$$

$$E = 2\pi k\sigma \left[1 - \frac{X}{\sqrt{R^2+X^2}}\right]$$

$\sigma = \frac{Q}{\pi R^2}$

## 5) Semicircle :



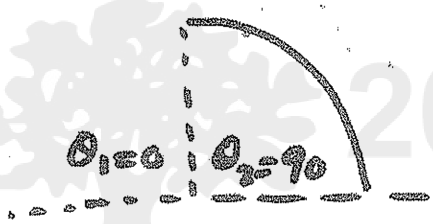

$Q = \lambda L$

$$\boxed{Q = \lambda \pi R}$$

$$\boxed{E_c = \frac{k\lambda}{R}\left[\sin\theta_1 + \sin\theta_2\right]}$$


$\theta_1 = 90$

$\theta_2 = 90$

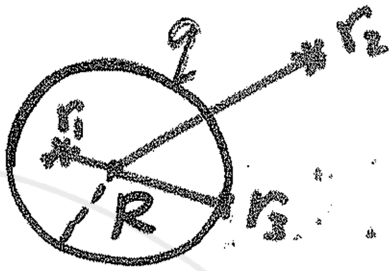



$$\Rightarrow \frac{k\lambda}{R}[0+1]$$


$\Rightarrow \theta_1 = 60$

$\theta_2 = 60$

[6] Conducting sphere
Spherical shell.




- at $r_1 < R$

$$\boxed{E_{in} = 0}$$

- at $r_2 > R$

$$q = \sigma (4\pi R^2)$$

$$E_{out} = \frac{kq}{r^2} = \frac{k\sigma 4\pi R^2}{r^2}$$

$$= \frac{\sigma R^2}{\epsilon_0 r^2}$$

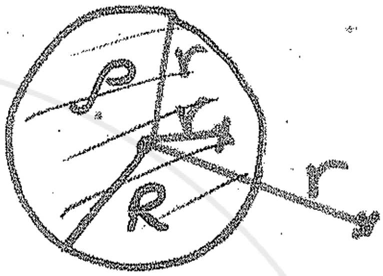
- at $r_3 = R$

$$\boxed{E = \frac{kq}{R^2} = \frac{\sigma}{\epsilon_0}}$$

# 7 Solid insulator sphere

ليست فارغة عازلة

① $r < R$ (in)




$$\boxed{E_{in} = \frac{kqr}{R^3} = \frac{\rho r}{3\epsilon_0}}$$

$$q = \rho\left(\frac{4}{3}\pi R^3\right)$$



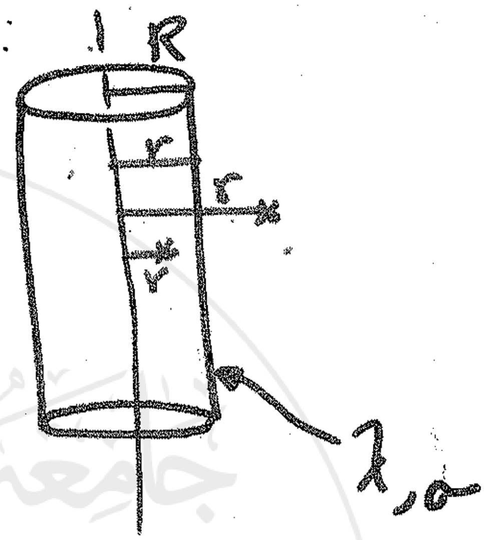
② $r > R$ (out)

$$E_{out} = \frac{kq}{r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

③ $r = R$

$$E = \frac{\rho R}{3\epsilon_0}$$

# [8] Conducting infinite cylinder
## cylindrical shell.




1) $r \leq R$ (in)

$$\boxed{E_{in} = 0}$$

---

2) $r > R$ (out)

$$E_{out} = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

$E(2\pi r) = \frac{\lambda L}{\epsilon}$

---

3) $r = R$

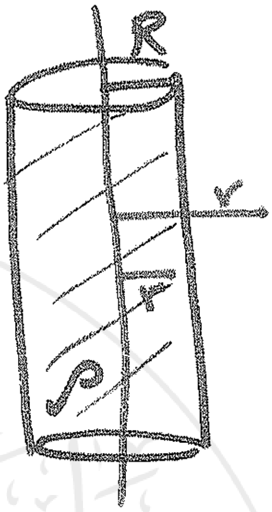
$$E_{sur} = \frac{\lambda}{2\pi\epsilon_0 R}$$

$$Q = Q$$

$$\sigma = \frac{\lambda}{2\pi R} \Leftarrow \lambda\cancel{L} = \sigma(2\pi R\cancel{L})$$

$$\Rightarrow E_{sur} = \frac{\sigma}{\epsilon_0} \quad , \quad E_{out} = \frac{R\sigma}{\epsilon_0 r}$$

## 9 Infinite Solid insulating cylinder




1) $r < R$ (in)

$$\boxed{E_{in} = \frac{\rho r}{2\epsilon_0}}$$

$E(2\pi r l)=\frac{\rho \pi r^2 l}{\epsilon_0}$

$E = \frac{\rho r}{2\epsilon_0}$

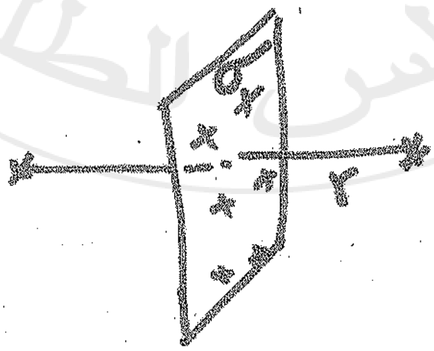
2) $r > R$

$$\boxed{E_{out} = \frac{\rho R^2}{2\epsilon_0 r}}$$

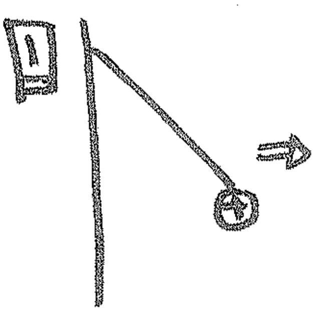
3) $r = R$

$$\boxed{E = \frac{\rho R}{2\epsilon_0}}$$

## 10 Infinite plate



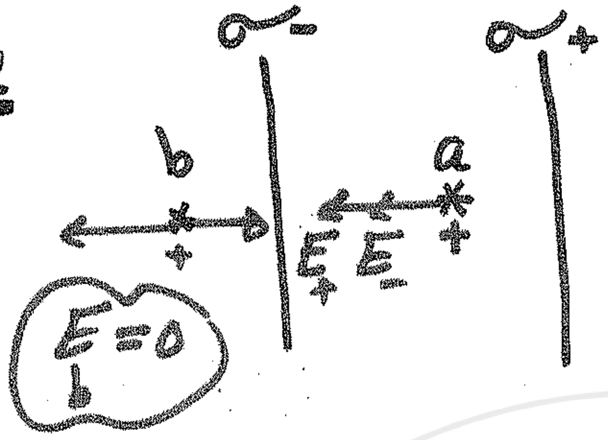

$\Rightarrow E_{\text{صفيحة}} = \frac{\sigma}{2\epsilon_0}$



$\Rightarrow$ find $E$ | find $\sigma$

$E = \checkmark$ | $E = \frac{\sigma}{2\epsilon_0} \Rightarrow \sigma = 2\epsilon_0 E$

**2**

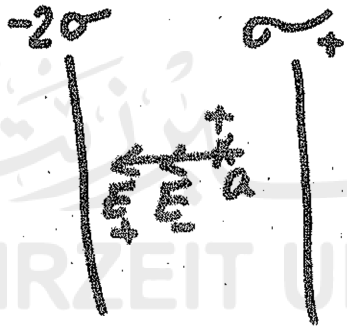



$$E_+ = \frac{\sigma}{2\epsilon_0}$$

$$E_- = \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E_a = E_+ + E_- = \frac{\sigma}{\epsilon_0}}$$

**3**




$$E_+ = \frac{\sigma}{2\epsilon_0}$$

$$E_- = \frac{2\sigma}{2\epsilon_0}$$

$$E_a = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$$

$\boxed{1}$

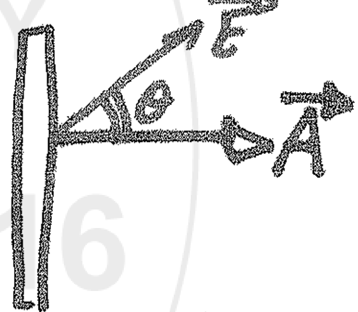
# CH: ~~24~~ 21 Electrical Flux and Gauss's law

**Flux**

$\Phi$ : عدد خطوط المجال التي تخترق سطحاً ما عمودياً عليه.

$(\Phi)$

1) سطح منتظم يخترقه مجال منتظم




$$\Phi = \vec{E} \cdot \vec{A}$$
$$= E A \cos\theta_{EA}$$

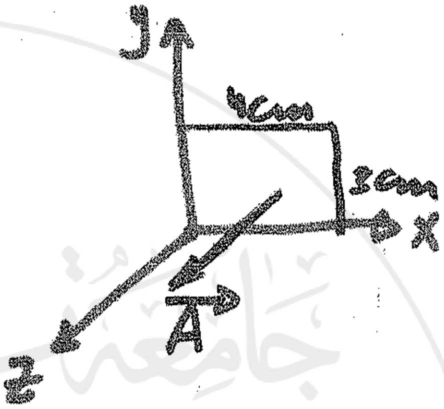
2) سطح مغلق بداخله شحنات


$$\Phi = \frac{\sum q_{in}}{\epsilon_0}$$
$$= \frac{q_1 + q_2}{\epsilon_0}$$

* السالب يعوض.

**Ex:** If $\vec{E} = 4\hat{i} + 2\hat{j} + 5\hat{k}$ and the Area is rectangular with dim. 4cm x 3cm lying on the xy ~~axis~~ plane. Find the electrical flux.

**Sol:**




$$A = 4 \times 10^{-2} \times 3 \times 10^{-2}$$

$$A = 12 \times 10^{-4} \ m^2$$

$$\vec{A} = 12 \times 10^{-4} \ \hat{k}$$

$$\Phi = \vec{E} \cdot \vec{A}$$

$$= (4\hat{i} + 2\hat{j} + 5\hat{k}) \cdot (12 \times 10^{-4} \ \hat{k})$$

$$= 60 \times 10^{-4} \ N \cdot m^2/C.$$

---

**Ex:** If the $\vec{E}$ is given by: $\vec{E} = 4\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{A}$ is given by: $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$, Find

1) The net flux

$$\Phi = \vec{E} \cdot \vec{A}$$

$$= 12 + 2 - 2$$

$$= 12 \ N \cdot m^2/C$$

2) angle between $\vec{E}$ and $\vec{A}$

$$\vec{E} \cdot \vec{A} = E A \cos\theta_{EA}$$

$$12 = \sqrt{16+1+4} \ \sqrt{9+4+1} \ \cos\theta$$

$$\cos\theta = \frac{12}{\sqrt{21} \times \sqrt{14}} = 0.7$$

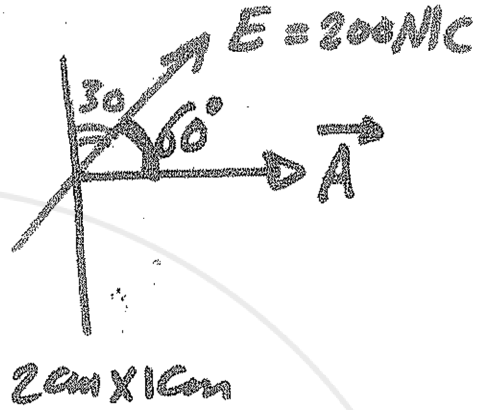
$$\boxed{\theta = 45.60}$$

**Ex**: In the figure, calculate the net flux

$\Phi = EA\cos\theta_{EA}$

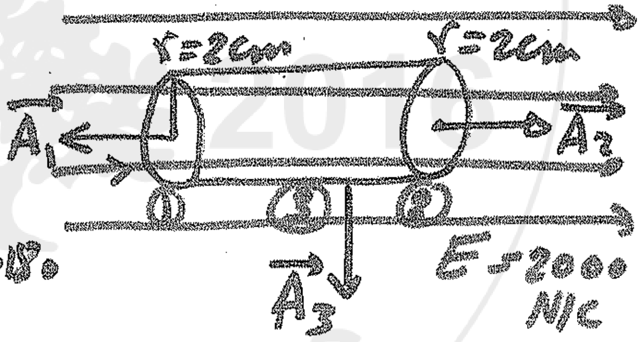
$= 200 \times (2\times10^{-4})\cos 60$

$= 0.02\ N\cdot m^2/C$.




**Ex**: Find the net Electrical Flux therough the below cylinder:

**Sol**




$\Phi_1 = EA\cos\theta$

$= 2\times10^{3} \times \pi(2\times10^{-2})^2 \cos 180$

$= -8\pi\times10^{-1}$

$= -0.8\pi\ N\cdot m^2/C$

$\Phi_2 = EA\cos\theta$

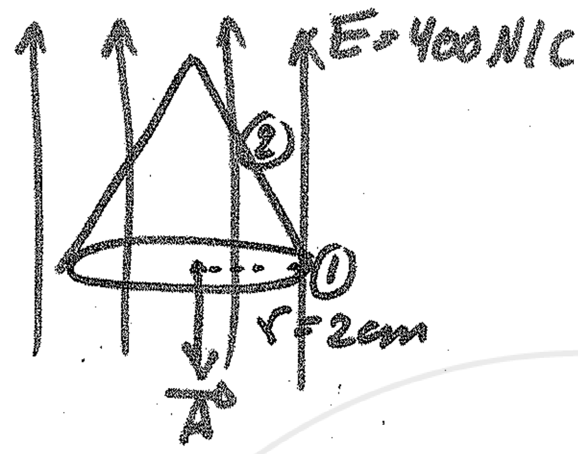
$= 2\times10^{3}\ \pi(2\times10^{-2})^2 \cos 0$

$= +0.8\pi\ N\cdot m^2/C$.

$\Phi_3 = 2\times10^{3} \times A_3 \cos 90 = 0$

$\Phi_{net} = -0.8\pi + 0.8\pi + 0 = 0$

**Ex:**




Find the Flux through each surface.

$$\Phi_1 = EA\cos\theta = 4\times10^{2} \times \pi(2\times10^{-2})^{2}\cos 180$$

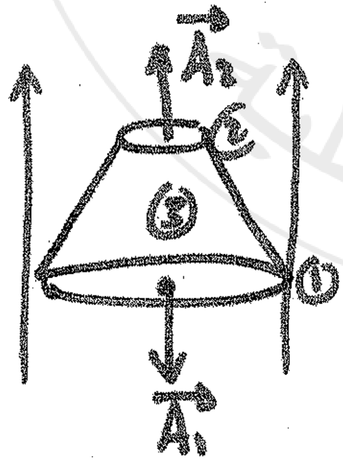
$$= -16\pi\times10^{-2} = -0.16\,\pi \text{ N}\cdot\text{m}^2/\text{C}$$

$$\Phi_{net} = \Phi_1 + \Phi_2$$

$$0 = -0.16 + \Phi_2 \Rightarrow \Phi_2 = +0.16 \text{ N m}^2/\text{C}.$$



**Ex**




$$\Phi_1 = \ldots \cos 180 = -a$$

$$\Phi_2 = EA_2\cos 0 = +b$$

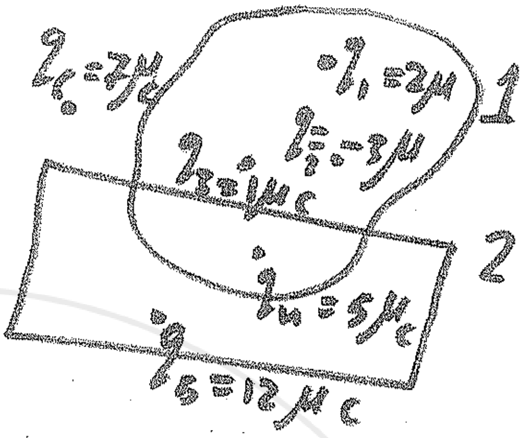
$$\Phi_3 = ??$$

$$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3$$

$$0 = -(a) + (b) + \Phi_3$$

$$\boxed{\Phi_3 = -b + a}.$$

**Ex:** Find flux through each surface.




$$\Phi_1 = \frac{\sum q_{ins}}{\epsilon_0}$$

$$= \frac{2\mu + -3\mu + 1\mu + 5\mu}{\epsilon_0}$$

$$= \frac{5\times10^{-6}}{8.85\times10^{-12}} = 0.56\times10^{6}\ N\cdot m^2/C.$$

$$\Phi_2 = \frac{5\mu + 12\mu}{\epsilon_0} = \frac{17\times10^{-6}}{8.85\times10^{-12}} = 1.92\times10^{6}\ N\cdot m^2/C.$$

---

**Ex:** If we have two charges, $q_1 = 6\mu C$ at the origin, $q_2 = -4\mu C$ at $x = 2$ cm. Find the net flux through a sphere of radius $r = 1$cm, centered at the origin.

$$\Phi = \frac{\sum q_{ins}}{\epsilon_0}$$

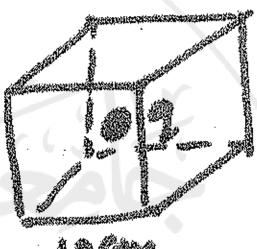
$$= \frac{6\times10^{-6}}{8.85\times10^{-12}} = 0.68\times10^{6}\ N\cdot m^2/C.$$

**Ex**: Cube of side 10 cm, Contains a charge at its center $q = 12\mu C$, Find:-

1) net flux.
2) Flux through each surface

1) $\Phi_{cube} = \dfrac{q}{\epsilon_0} = \dfrac{12 \times 10^{-6}}{8.85 \times 10^{-12}}$

$= 1.4 \times 10^{6}\ N.m^2/C.$




2) $\Phi_{face} = \dfrac{\Phi_{net}}{6} = 0.23 \times 10^{6}\ N.m^2/C.$



---

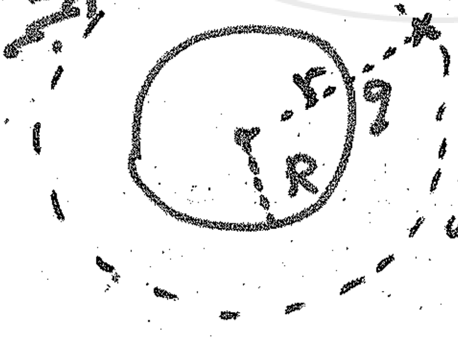
$$EA = \frac{\sum q_{ins}}{\epsilon_0} \Rightarrow \int E\,dA = \frac{q_{ins}}{\epsilon_0}$$

المجال الكهربائي ثابت

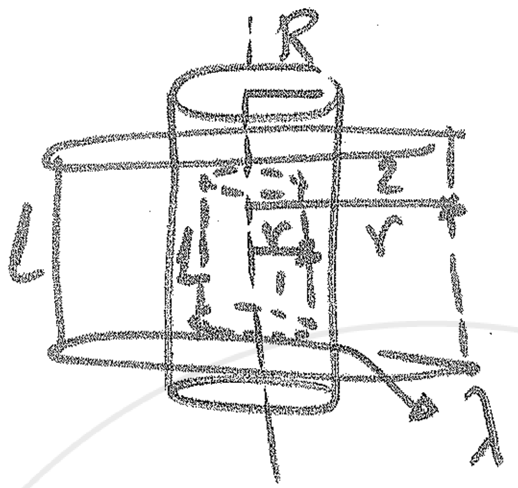
مساحة سطح غاوس المحيط بالجسم

الشحنة الكلية داخل سطح غاوس

$$q_{ins} = \begin{cases} \lambda L \rightarrow \int \lambda\,dx \\ \sigma A \rightarrow \int \sigma\,dA \\ \rho V \rightarrow \int \rho\,dV \\ q \\ 0 \end{cases}$$



Ex :




1) $EA = \frac{q_{ins}}{\epsilon_0}$

$E(2\pi r l) = \frac{0}{\epsilon_0}$

$E_{in} = 0$

2) $r > R$

$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\epsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

Ex


1) $r < R$

$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(4\pi \cancel{r^2}) = \frac{\rho \frac{4}{3}\pi r^{3}}{\epsilon_0}$

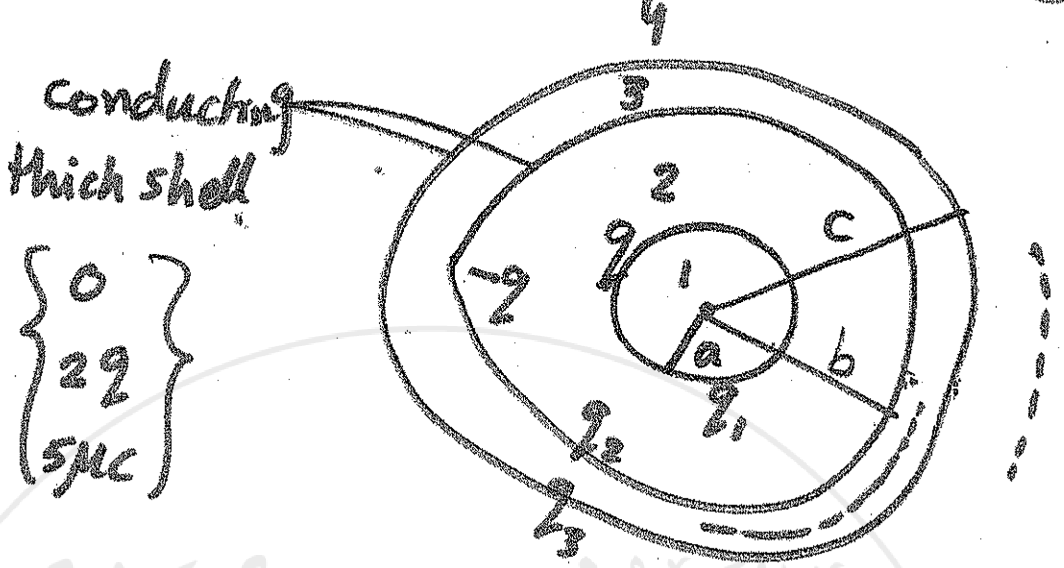
$E = \frac{\rho r}{3\epsilon_0}$

2) $r > R$

$EA = \frac{q_{ins}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi R^3)}{\epsilon_0}$

$E = \frac{\rho R^3}{3\epsilon_0 r^2}$

Ex:




$$q_{shell} = \begin{Bmatrix} 0 \\ 2q \\ 5\mu C \end{Bmatrix}$$

| | | |
|---|---|---|
| $q_{shell} = q_2 + q_3$ | $q_{shell} = q_2 + q_3$ | $\boxed{q_2 = -q_1}$ |
| $0 = -q + q_3$ | $2q = -q + q_3$ | |
| $\boxed{q_3 = q}$ | $q_3 = 3q$ | |

Find $\underline{E}$ every where.

① $r < a$ : $E_1 = \dfrac{\rho r}{3\epsilon_0}$ (عازل) | $E_1 = 0$

② $a < r < b$ : $E_2 = \dfrac{kq}{r^2} \Rightarrow E_2 = \dfrac{\rho R^2}{3\epsilon_0 r^2}$

③ $c > r > b$ : $EA = \dfrac{q_{ins}}{\epsilon_0}$

$$E(4\pi r^2) = \frac{(0)}{\epsilon_0}$$

$$E_3 = 0$$

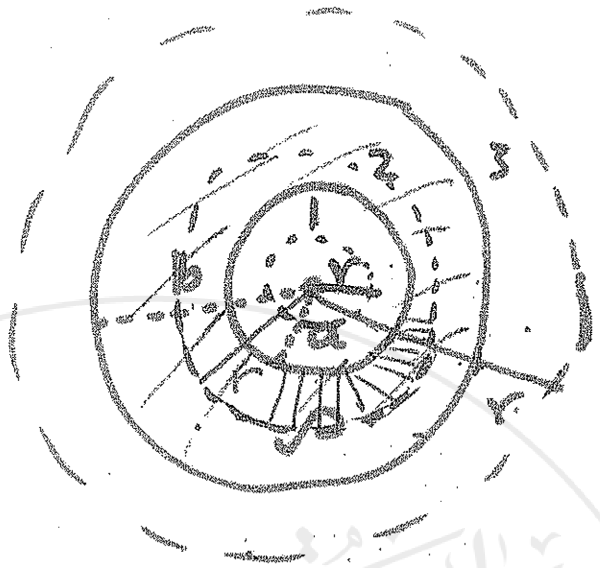
④ $r > c$

$$E(4\pi r^2) = \frac{q_3}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Ex






1) $r<a$

$$EA=\frac{q_{ins}}{\epsilon_0}\Rightarrow E(4\pi r^2)=\frac{0}{\epsilon_0}\Rightarrow E_1=0$$

2) $a<r<b$

$$EA=\frac{q_{ins}}{\epsilon_0}\Rightarrow E(4\pi r^2)=\frac{\rho\left(\frac{4}{3}\pi r^3-\frac{4}{3}\pi a^3\right)}{\epsilon_0}$$

$$E=\frac{\rho(r^3-a^3)}{3\epsilon_0 r^2}=\frac{\rho\left(r-\frac{a^3}{r^2}\right)}{3\epsilon_0}$$

3) $r>b$

$$EA=\frac{q_{ins}}{\epsilon_0}\Rightarrow E(4\pi r^2)=\frac{\rho\left(\frac{4}{3}\pi b^3-\frac{4}{3}\pi a^3\right)}{\epsilon_0}$$

$$\boxed{E_3=\frac{\rho(b^3-a^3)}{3\epsilon_0 r^2}}$$

# CH: ~~25~~ 22 Electrical Potential

Volt ← (V) الجهد الكهربائي


Potential difference between a and b.

$\Rightarrow V_{ab} = V_a - V_b$

U : Potential energy

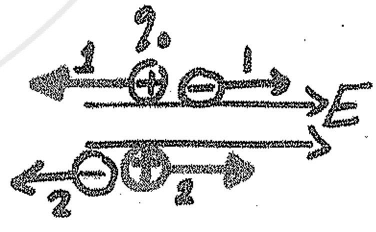
$$\left.\begin{array}{l} U_a = q_0 V_a \\ U_b = q_0 V_b \end{array}\right] \Rightarrow \Delta U_{a\to b} = U_b - U_a = q_0 [V_b - V_a]$$
$$= q_0 \Delta V_{a\to b}$$
$$= q_0 V_{ba}$$

## $\boxed{1}$ V due to Point charge

$$W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} = q_0 V_{ba} \quad [\Delta K = 0]$$

## $\boxed{2}$ V due to Uniform E field:

① $W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} \quad [\Delta K = 0]$



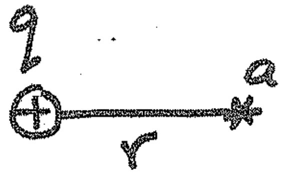

② $W_{a\to b} = \underset{\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)}{\Delta K} = -\Delta U_{a\to b} = -q_0 \Delta V_{a\to b}$

## $\boxed{3}$ V due to dist. of charges:

$$W_{a\to b} = \Delta U_{a\to b} = q_0 \Delta V_{a\to b} \quad [\Delta K = 0]$$

# 1 Electrical Potential due to Point charge 2

$$V = \frac{kq}{r}$$



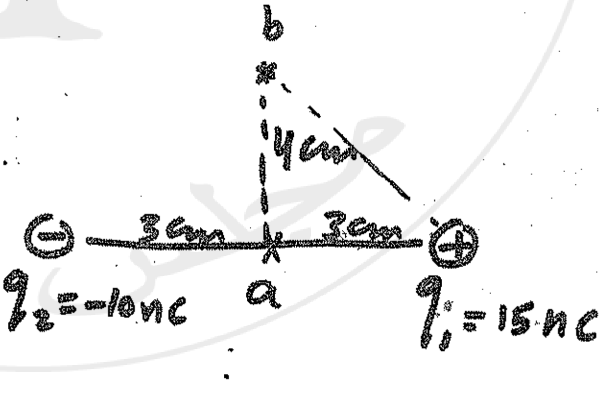

* السالب تعوض.
* الجهد ليس له اتجاه.
* لحساب الجهد الكلي: $V_{total} = V_1 + V_2 + V_3 + \dots$

---

**Ex:** In the figure, Find:

1) Potential difference between a and b.
2) Work needed to bring $q_0 = 2\mu C$ from a to b.
3) " " " " " a to $\infty$

**Sol**




1) $V_a = V_1 + V_2$

$$= \frac{9\times10^{9} \times 15\times10^{-9}}{3\times10^{-2}} + \frac{9\times10^{9} \times -10\times10^{-9}}{3\times10^{-2}}$$

$$V_a = 45\times10^{2} - 30\times10^{2} = 15\times10^{2} \text{ Volt}.$$

$$V_b = \frac{9\times10^{9} \times 15\times10^{-9}}{5\times10^{-2}} + \frac{9\times10^{9} \times -10\times10^{-9}}{5\times10^{-2}} = (27-18)\times10^{2}$$

$$= 9\times10^{2} \text{ Volt}$$

$$V_{ab} = V_a - V_b$$

$$= 15\times10^{2} - 9\times10^{2} = 6*10^{2} \text{ Volt.}$$

---

$$\Delta V_{a\to b} = V_{ba} = -V_{ab} = -6*10^{2} \text{ Volt.}$$

---

2) $$W_{a\to b} = q_0 \, \Delta V_{a\to b}$$

$$= 2*10^{-6} * [-6*10^{2}]$$

$$= -12*10^{-4} \text{ J.}$$

---

3) $$W_{a\to\infty} = q_0 \, [V_\infty - V_a]$$

$$= 2*10^{-6} \, [0 - 15\times10^{2}]$$

$$= -30*10^{-4} \text{ J.}$$

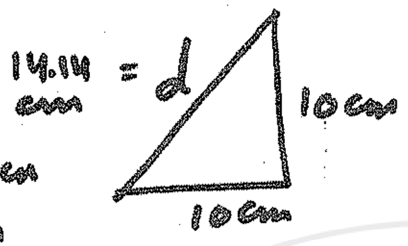
---

$\boxed{4}$ What is the potential at the Position of $q_1$.

$$V_{at\,q_1} = \frac{9*10^{9} * -10*10^{-9}}{6*10^{-2}} = -15*10^{2} \text{ Volt.}$$

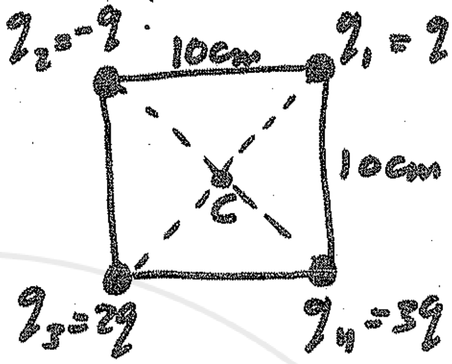
**Ex**: Find the Electrical Potential at $\underline{\underline{c}}$

$(q = 7nc)$




$\frac{1}{2}d = 7.07cm$

$\simeq 7cm$




$$V_c = V_1 + V_2 + V_3 + V_4$$

$$= \cancel{\frac{9*10^9 * 7*10^{-9}}{7*10^{-2}}} + \cancel{\frac{9*10^9 * -7*10^{-9}}{7*10^{-2}}} + \frac{9\times10^9 \times 14*10^{-9}}{7\times10^{-2}}$$

$$+ \frac{9\times10^9 * 21*10^{-9}}{7\times10^{-2}}$$

$$V_c = \frac{9\times10^9 \times 10^{-9}}{7\times10^{-2}}[14 + 21] = 45*10^2 \text{ Volt}$$

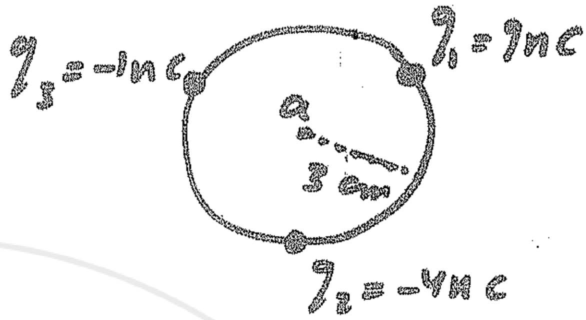
b) If we want the Work to bring an $\underset{\sim}{\text{electron}}$ from $\infty$ to C: $q_e = -1.6*10^{-19}c$

$$W = q_0 [V_c - V_\infty]$$

$$= -1.6\times10^{-19} [45\times10^2 - 0]$$

$$= -72*10^{-17} J.$$

Ex: what is the Work needed to bring a proton from $\infty$ to $\underline{a}$.




$$\Rightarrow W = q_0 [V_a - \cancel{V_\infty}]$$

$$= +1.6 \times 10^{-19} [V_a]$$

$$= 1.6 \times 10^{-19} \times 12 \times 10^{-2} \quad = 19.2 \times 10^{-21} \text{ J}.$$

$$V_a = V_1 + V_2 + V_3$$

$$= \frac{9 \times 10^{9}}{3 \times 10^{-2}} [9 \times 10^{-9} + -4 \times 10^{-9} + -1 \times 10^{-9}]$$

$$= 3 \times 10^{7} [4 \times 10^{-9}] = 12 \times 10^{-2} \text{ Volt}.$$

---

* Energy stored in فقط $\underline{\underline{q_1}}$ :


$$U_1 = U_{12} + U_{13}$$

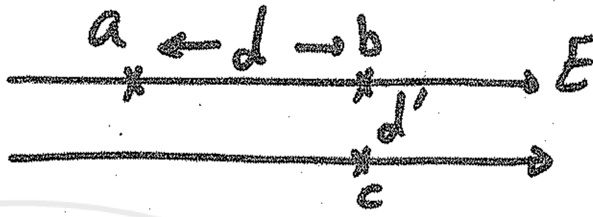
$$= \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}}$$

* Energy stored in the system :

$$U_{sys} = U_{12} + U_{13} + U_{23} \longrightarrow \frac{k q_2 q_3}{r_{23}}$$

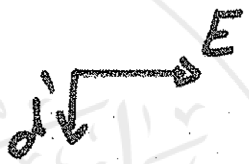
## [2] Electric Potential due Uniform $\underline{\underline{E}}$:

$$V_{ba} = \Delta V_{a\to b} = -E d \underset{Ed}{\cos\theta}$$




$$= -\vec{E}\cdot\vec{d}$$

* $\Delta V_{b\to c} = -E(d')\cos 90$




$\Delta V_{b\to c} = 0$

$V_c - V_b = 0 \Rightarrow \boxed{V_c = V_b}$

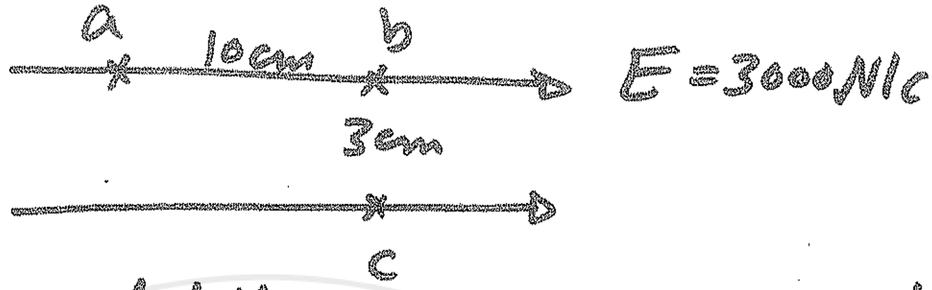
* $\Delta V_{a\to c} = \Delta V_{a\to b} + \cancel{\Delta V_{b\to c}}$

* if we know $V_a \Rightarrow$ what is $V_b$:

$\Delta V_{a\to b} = \checkmark \Rightarrow V_b - V_a = \checkmark$

$V_b = V_a + \checkmark$ .

Ex:




1) Find Potential difference between a and b.
b " c
a " c.

2) Find the Work needed to move $q_0 = 2\mu C$ from a to b.

3) if $q_0$ in Part(2) started from rest ($m = 2gm$), what is its final speed.

4) Find the Work needed to move $q_0 = 5\mu C$ from b to a.

5) Find the Work needed to move $q_0 = -1\mu C$ from a to b.

6) if $V_a = 20$ Volt, what is $V_b$.

7*) what is the Work needed to move $q_0$ from $a \to a$ along the path $a \to b \to c \to a$

**Sol :**

1) $$V_{ab} = \Delta V_{b\to a} = -Ed\cos\theta_{a\to b}$$

$$= -3\times10^{3} * 10\times10^{-2} * \cos 180$$

$$= 300 \text{ Volt}.$$

* $$\Delta V_{a\to b} = -300 \text{ Volt}.$$

2) * $$\Delta V_{c\to b} = 0$$

* $$\Delta V_{ac} \Rightarrow \Delta V_{c\to a} = \cancel{\Delta V_{c\to b}} + \Delta V_{b\to a}$$

$$= 300 \text{ Volt}.$$

2) $$W = -q_0 \Delta V_{a\to b}$$

$$= -2\times10^{-6}[-300] = 600\times10^{-6}\,J$$

3) $$\Delta K_{a\to b} = -q_0 \Delta V_{a\to b}$$

$$\tfrac{1}{2}mV_b^2 - \cancel{\tfrac{1}{2}mV_a^2} = 600\times10^{-6}$$

$$\tfrac{1}{\cancel{2}} * \cancel{2}\times10^{-3} * V_b^2 = 600\times10^{-6} \Rightarrow V_b = 0.77\,m/s.$$

(4) $W_{b\to a} = q_o \Delta V_{b\to a}$

$= 5 * 10^{-6} [300]$

$= 1500 * 10^{-6} \, J$.

(5) $W_{a\to b} = q_o \Delta V_{a\to b}$

$= -1 * 10^{-9} * -300$

$= 300 * 10^{-9} \, J$.

(6) $\Delta V_{a\to b} = -300$

$V_b - V_a = -300$

$V_b - 20 = -300 \implies V_b = -280$ Volt.

(7) $W_{a\to a} = q_o \Delta V_{a\to a}$

$= q_o [\Delta V_{a\to b} + \Delta V_{b\to c} + \Delta V_{c\to a}]$

$= q_o [-300 + 0 + 300]$

$=$ Zero.

## [3] Potential due to distribution of charges (10)

الجهد الناتج عن توزيع من الشحنات

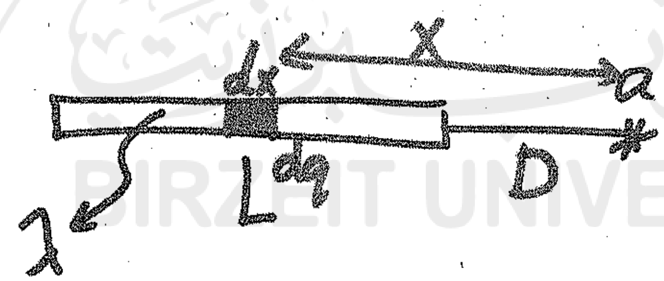
$$q = \lambda L \quad , \quad q = \sigma A \quad , \quad q = \rho V$$

$$dq = \lambda dx$$

---

**Ex:**

$q = \lambda L$




$$V = \frac{kq}{r} \Rightarrow dV = \int \frac{k\,dq}{r} = \int \frac{k\lambda\, dx}{r = x}$$

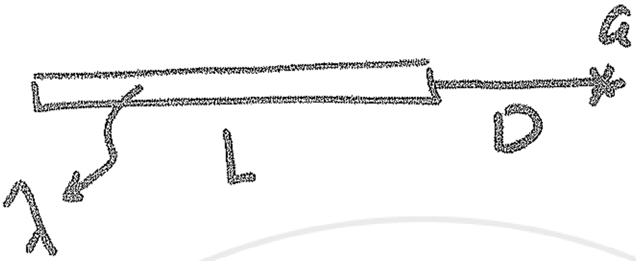
$$dV = k\lambda \int_{D}^{D+L} \frac{dx}{x} = k\lambda \ln x \Big|_{D}^{D+L}$$

$$= k\lambda \left[\ln(D+L) - \ln D\right]$$

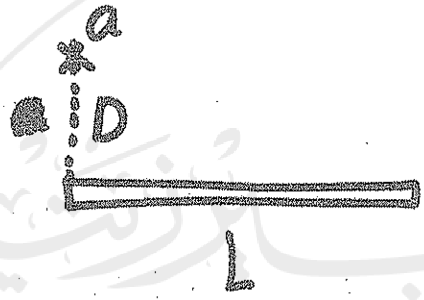
$$= k\lambda \ln\left(\frac{D+L}{D}\right)$$

① **rod:**

$q = \lambda L$




$$V_a = k\lambda \ln\left(\frac{D+L}{D}\right)$$

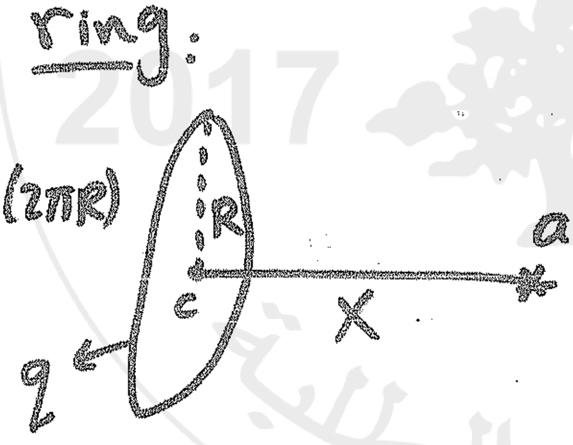



$$\Rightarrow V_a = k\lambda \ln\left[\frac{L+\sqrt{D^2+L^2}}{D}\right]$$

---

[2] **ring:**

$q = \lambda(2\pi R)$



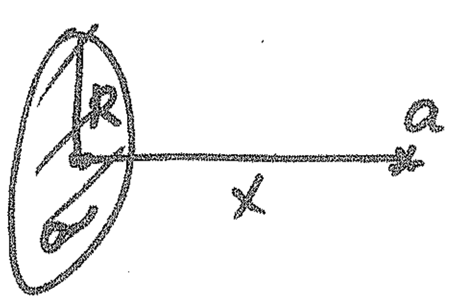

$$V_a = \frac{kq}{\sqrt{R^2+x^2}}$$

$$V_c = \frac{kq}{R}$$

---

[3] **Disk**

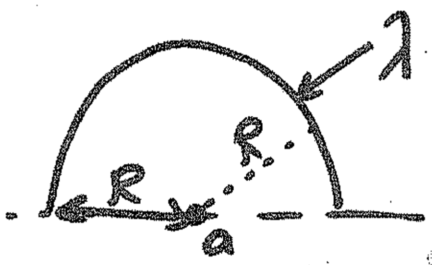
$$V_a = 2\pi \underset{9\times10^9}{k}\sigma\left[\sqrt{R^2+x^2} - x\right]$$




$q = \sigma \pi R^2$

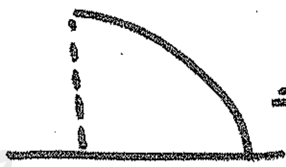
## [4] Semicircle:

$q = \lambda \pi R$




$V_a = k \lambda \theta$

$\theta \rightarrow 3.14 = \pi$



$\Rightarrow \theta = \frac{\pi}{2} \Rightarrow V = \frac{k\lambda\pi}{2}$


$\Rightarrow V_a = V_1 + V_2 + V_3$.

$= k\lambda \ln\left(\frac{D+l}{D}\right) + k\lambda\theta + k\lambda \ln\left(\frac{D+l}{D}\right)$

$= k\lambda\left(\ln\left(\frac{R+2R}{R}\right)\right) + k\lambda\pi + k\lambda \ln\left(\frac{R+3R}{R}\right)$

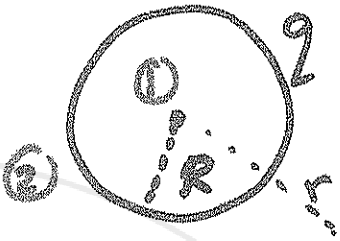
$= k\lambda \ln 3 + k\lambda\pi + k\lambda \ln 4$

= ( جمع )

## 5 Conducting sphere:-

1) $r<R$ (inside)

$$V_{in} = \frac{kq}{R} = V_{surface}$$




$$q = \sigma 4\pi R^2$$

2) $r>R$ (outside)

$$V_{out} = \frac{kq}{r}$$

---

## 6 insulating sphere


$$q = \rho \frac{4}{3}\pi R^3$$

1) $r<R \Rightarrow V_{in} = \frac{kq}{2R}\left(3 - \frac{r^2}{R^2}\right)$

2) $r \geq R \Rightarrow V_{out} = \frac{kq}{r}$

$$\Delta U = W = q_0 \Delta V$$

(14) إذا اعطاك في السؤال $\vec{E}$ (المعادلة) وطلب $V$ :

$$\boxed{\Delta V = -\int E_i\,dx - \int E_j\,dy - \int E_k\,dz - \int E\,dr}$$

**Ex**: If $E = \frac{4}{r^2}$ , Find $V$ from $\infty$ to 5cm

sol

$$\Rightarrow V = -\int E\,dr$$

$$= -\int_{\infty}^{5\,cm} \frac{4}{r^2}\,dr = -4\left[-\frac{1}{r}\right]_{\infty}^{5\,cm}$$

$$= \frac{4}{5\times10^{-2}} - 0 = 80 \text{ Volt.}$$

---

Ex: If $\vec{E} = 3x^2yz\,\hat{i} - 5xz\,\hat{j} + 2zy\,\hat{k}$

Find $V$

$$V = -\int 3x^2yz\,dx - \int -5xz\,dy - \int 2zy\,dz$$

$$= -3yz\frac{x^3}{3} + 5xzy - 2y\frac{z^2}{2}$$

$$= -yzx^3 + 5xyz - yz^2 .$$

$(1, 1, -2)$

(15) إذا أعطاك $V$ على شكل معادلة وطلب $\vec{E}$ :

$$\vec{E} = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z} - \hat{r}\frac{\partial V}{\partial r}$$

**Ex**: If $V = 2x^2yz^3 + 5xz$, Find:-

1) $\vec{E}$ at $(1,2,-1)$
2) magnitude of $\vec{E}$ at $(1,2,-1)$
3) if $q_0 = 2\mu C$, Find Force.

**Sol**:

1) $\frac{\partial V}{\partial x} = 4xyz^3 + 5z \quad \Big| \quad \frac{\partial V}{\partial y} = 2x^2z^3 \quad \Big| \quad \frac{\partial V}{\partial z} = 6x^2yz^2 + 5x$

$\Rightarrow \frac{\partial V}{\partial x} = -8 + -5 = -13$

$\frac{\partial V}{\partial y} = -2 \quad \Big| \quad \frac{\partial V}{\partial z} = 12 + 5 = 17$

$\Rightarrow \vec{E} = -(-13)\hat{i} - (-2)\hat{j} - (17)\hat{k}$

$$\boxed{\vec{E} = 13\hat{i} + 2\hat{j} - 17\hat{k}}$$

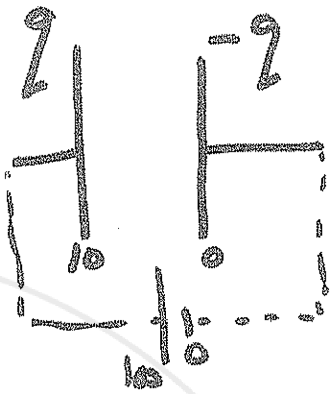
2) $|\vec{E}| = \sqrt{13^2 + 2^2 + 17^2}$ N/C

3) $\vec{F} = q_0\vec{E} = 2\times10^{-6}(13\hat{i} + 2\hat{j} - 17\hat{k})$

# CH: ~~26~~ 23 Capacitors ①

$C \equiv$ Capacitance

المواسعة الكهربائية



$$\boxed{C = \frac{q}{\Delta V}}$$

عام

$\Delta V = V_{+} - V_{-}$

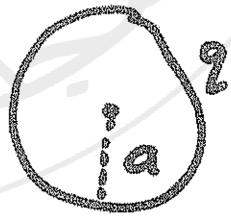
هو أي جسمين متلاصقين يصبح لهما شحنات أكبر

Farad

## ① spherical capacitor:

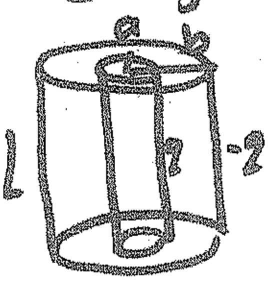
$$\boxed{C = \frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

## ② sphere:



$$\boxed{C = 4\pi\epsilon_0 a}$$

## ③ cylindrical capacitor

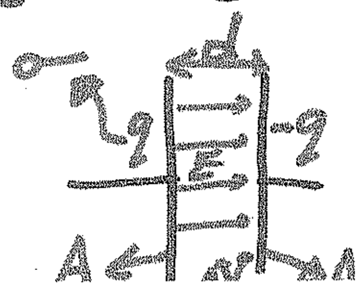


$$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

## ④ Two Parallel plates Capa

مكثف ذو لوحين متوازيين

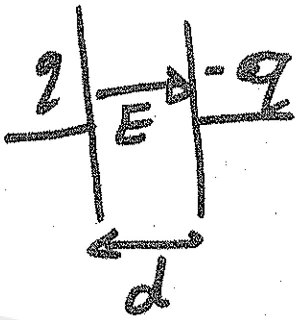
$$C = \frac{\epsilon_0 A}{d}$$



$C = \frac{\epsilon_0 A}{d}$ $\quad E = \frac{\sigma}{\epsilon_0}$

$q = \sigma A$

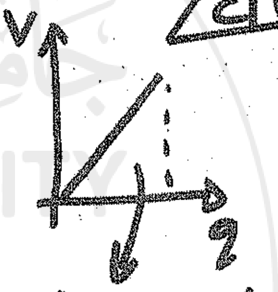
$C = \frac{q}{\Delta V}$ لوحين $\quad \Delta V = E d$ لوحين




---

* Energy stored in capacitor :


$$\boxed{U = \frac{1}{2} q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{q^2}{C}}$$




Area = U

$W = \Delta U = U_f - U_i$

---

$u \equiv$ Energy density ( كثافة الطاقة )

energy per Unit Volume.

الطاقة في وحدة الحجوم.

$$\boxed{u = \frac{U}{\underbrace{\text{Volume}}_{Ad}} = \frac{1}{2} \epsilon_0 E^2}$$

Ex: Two Parallel Plates Capacitor has (3)

a surface charge density $(\sigma) = 2\times10^{-6}\ C/m^2$

and Area of $2\ cm^2$ with separation $10\ cm$,

$A = 2\times10^{-4}\ m^2$ البعد بين اللوحين $d = 10\times10^{-2}\ m$

Find:-

① Capacitance (c). $\Rightarrow C = \frac{\epsilon_0 A}{d}$

② charge (q). $\Rightarrow q = \sigma A$

③ Potential difference accross the Cap. $\Rightarrow V = \frac{q}{C}$

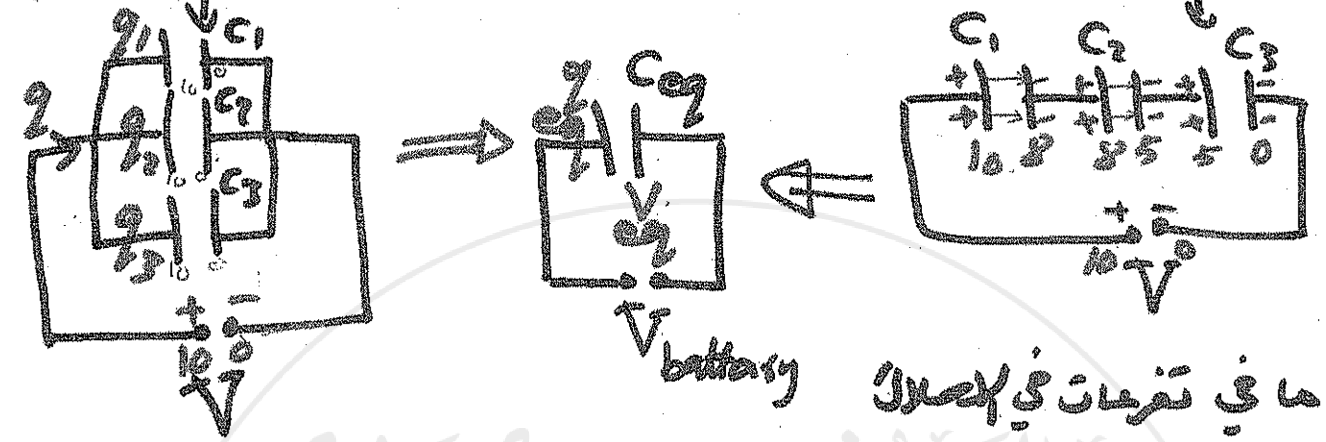
④ Electric field. $\Rightarrow E = \frac{\sigma}{\epsilon_0}$ or $E = \frac{V}{d}$

⑤ Energy stored in Cap. $\Rightarrow U = \frac{1}{2} q V = \frac{1}{2} C V^2$

⑥ energy density. (u) $\Rightarrow u = \frac{1}{2}\epsilon_0 E^2 = \frac{U}{Ad}$

# Connection of Capacitors (4)

(التوازي) in Parallel — (التوالي) in Series




- ما في تفرعات في الأسلاك

* اللوح الموجب متصل مع اللوح السالب

* تكون الشحنة متساوية

$$q_{eq} = q_1 = q_2 = q_3 \dots$$

* يتوزع الجهد الكهربائي

$$V_{eq} = V_{bat} = V_1 + V_2 + V_3 + \dots$$

* السعة مقلوبة

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

* يوجد تفرع في الأسلاك ولكن لكل تفرع مكثف واحد فقط والفروع لها نفس البداية والنهاية

* اللوح الموجب مع الموجب والسالب مع السالب

* تتوزع الشحنة الكهربائية:

$$q_{eq} = q_1 + q_2 + q_3 + \dots$$

* الجهد الكهربائي متساوي:

$$V_{bat} = V_{eq} = V_1 = V_2 = V_3 = \dots$$

* السعة مجموعة:

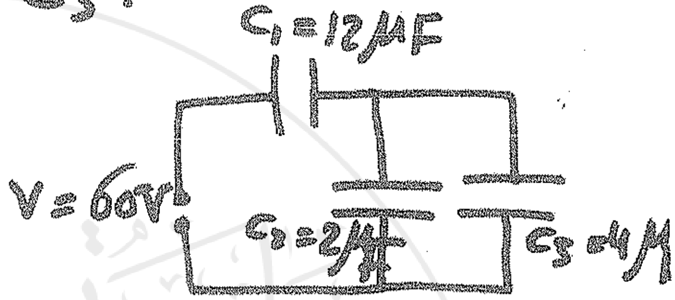
$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

Ex: In the figure shown, Find:

① equivalent capacitance

② $q$, $V$ for all capacitors.

③ energy stored in $C_5$.




**Sol:**

$C_2, C_3 \xrightarrow{\text{توازي}} C_4$

$C_1, C_4 \xrightarrow{\text{توالي}} C_5$

$V_5 = 60V$

* ارسم خريطة للسؤال وتعرف عليها وعلاقة بالإجابة

* نجد $C_{eq}$

* نجد جميع معلومات المطلوب النهائي

* من الخريطة نبدأ من المطلوب النهائي للوصول للمطلوب.

① $C_4 = C_2 + C_3$
$= 2\mu + 4\mu$
$= 6\mu F$

$$\frac{1}{C_5} = \frac{1}{C_1} + \frac{1}{C_4}$$

$$\frac{1}{C_5} = \frac{1}{12\mu} + \frac{1}{6\mu}$$

$$\boxed{C_5 = 4\mu F}$$

$\Downarrow$

$q_5 = C_5 V_5$

$\boxed{q_5 = 240\mu C}$

② $q_1 = q_4 = q_5 = 240\mu C$

$\longrightarrow V_4 = \frac{q_4}{C_4} = 40 \text{ Volt}$

$\longrightarrow V_1 = \frac{q_1}{C_1} = 20 \text{ Volt}.$

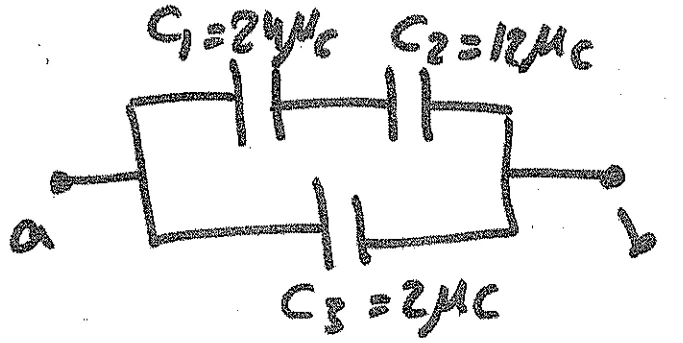
$V_2 = V_3 = V_4 = 40 \text{ Volt}$

$\longrightarrow q_3 = C_3 V_3 = 160\mu C$

$\longrightarrow q_2 = C_2 V_2 = 80\mu C$

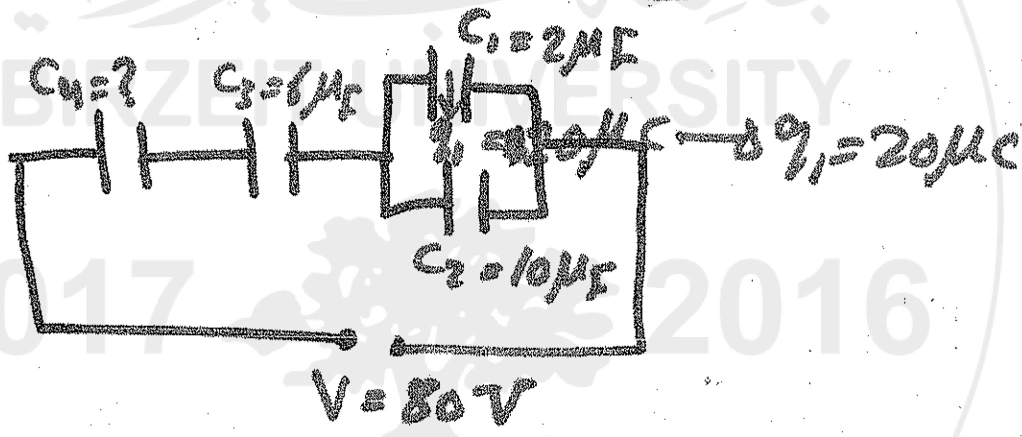
③ [illegible]

**Ex:**




$V_{ab} = 120V$

Find : $C_{eq}$ , $V$, $q$ for all $C$'s , $V_3$ .



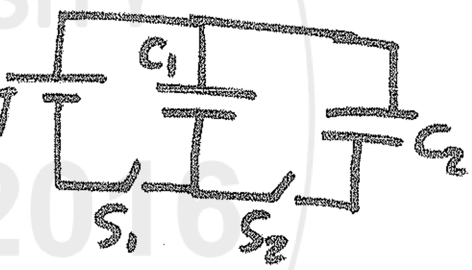

Find $C_4$ .

$q, V$ for all $C$'s.

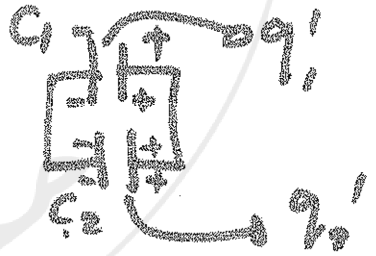
**Ex:** Capacitor has $C_1 = 6\mu F$, Connected to Voltage of 40 Volt. then disconnect from battary, and then Connected to anothe Capacitor $C_2 = 3\mu F$ initially Uncharged. Find <u>q</u> and <u>V</u> for $C_1$ and $C_2$ after Connected together.

**Sol:**

$$\left.\begin{matrix} C_1 = 6\mu F \\ V_1 = 40\ Volt \end{matrix}\right\} \Rightarrow q_1 = 240\mu C$$




$$\left.\begin{matrix} C_2 = 3\mu F \\ V_2 = 0 \end{matrix}\right\} \Rightarrow q_2 = 0$$




$$V_1' = V_2'$$

$$\frac{q_1'}{\cancel{6\mu}\,_2} = \frac{q_2'}{\cancel{3\mu}}$$

$$\Rightarrow \boxed{q_1' = 2q_2'}$$

$$\sum q_{bef} = \sum q'_{after}$$

$$q_1 + q_2 = q_1' + q_2'$$

$$240\mu + 0 = q_1' + q_2'$$

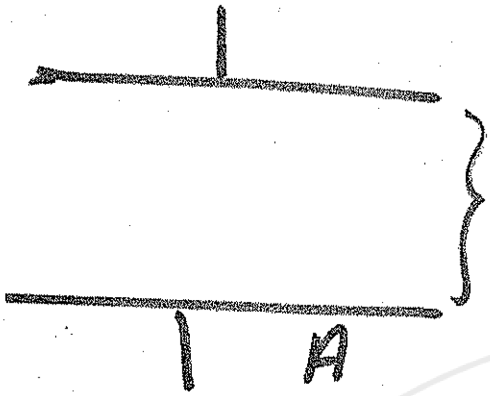
$$\Rightarrow 240\mu = 2q_2' + q_2' = 5q_2'$$

$$\Rightarrow \boxed{q_2' = 80\mu C}$$

$$\Rightarrow \boxed{q_1' = 160\mu C}$$

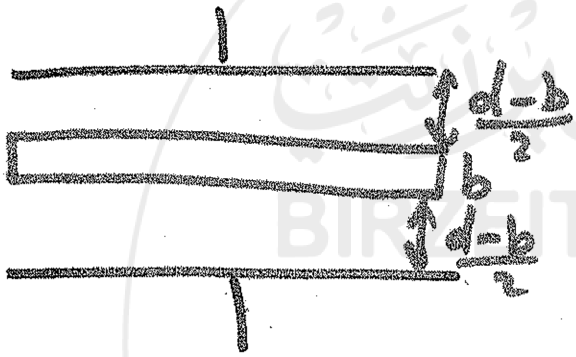
$$V_1' = \frac{160\mu}{6\mu} = 26.7\ V = V_2'$$

# metalic slap inside $C$




$$\Rightarrow C = \frac{\epsilon_0 A}{d}$$

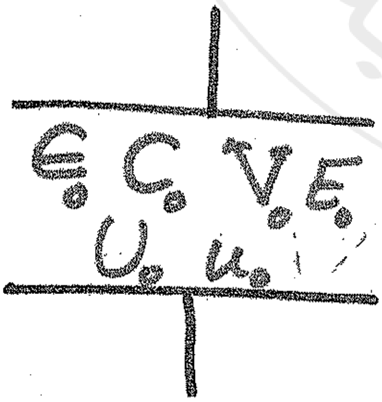
⇓




$$\Rightarrow C = \frac{\epsilon_0 A}{(d-b)}$$

Insulatic material
( Dielectric material)

$K$ : dielectric constant ($K = 1$ air
الهواء

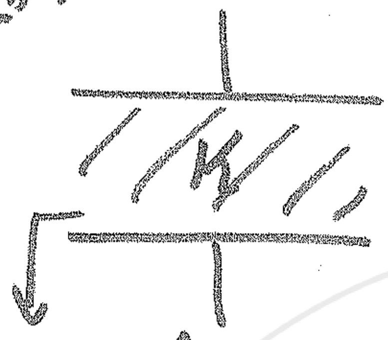



$\Rightarrow$


$\boxed{\epsilon = K\epsilon_0}$ $\boxed{C = KC_0}$

$\boxed{V = \frac{V_0}{K}}$ $\boxed{E = \frac{E_0}{K}}$ $\boxed{U = \frac{U_0}{K}}$ $\boxed{u = \frac{u_0}{K}}$

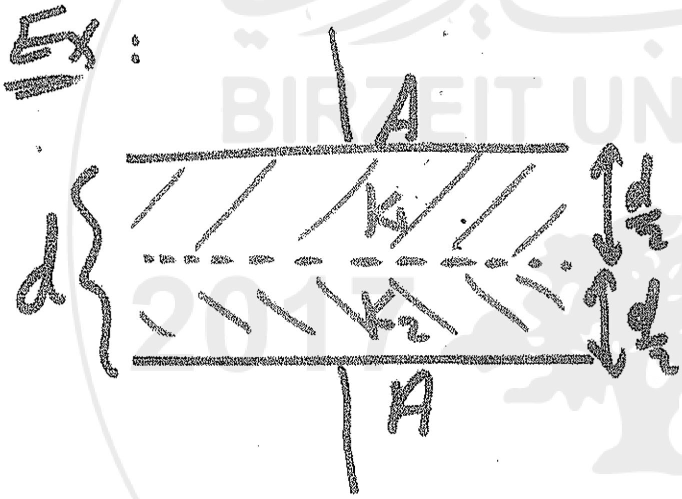
**Ex:**



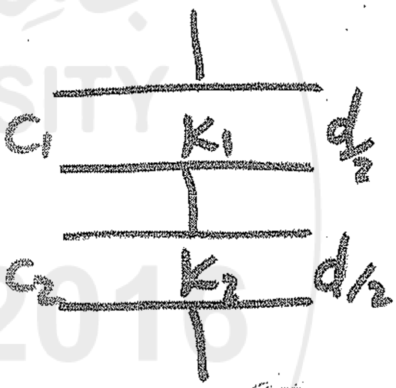

$$\Longrightarrow \quad C = \frac{K\epsilon_0 A}{d} = K C_0$$

$$C_0 = \frac{\epsilon_0 A}{d}$$

**Ex:**




$\Longrightarrow$




$$C_1 = \frac{2K_1\epsilon_0 A}{d} \quad C_0$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_2 = \frac{2K_2\epsilon_0 A}{d} \quad C_0$$

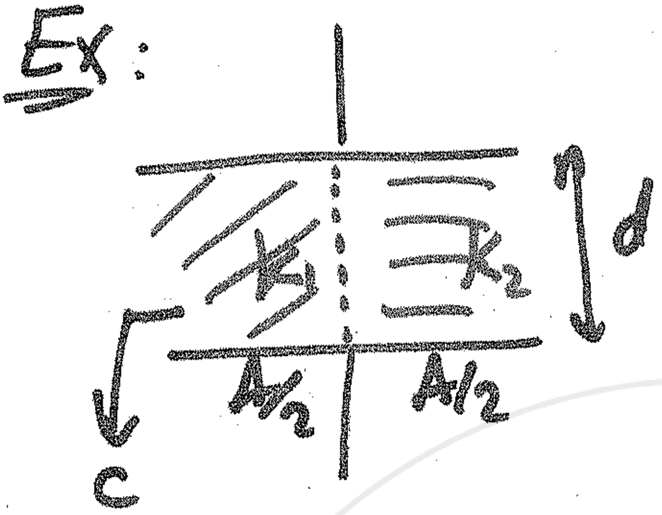
$$= \frac{d}{2K_1\epsilon_0 A} + \frac{d}{2K_2\epsilon_0 A}$$

$$\frac{1}{C_{eq}} = \frac{dK_2 + dK_1}{2K_1K_2\epsilon_0 A} \Rightarrow C_{eq} = \frac{2K_1K_2}{(K_1+K_2)}\frac{\epsilon_0 A}{d}$$

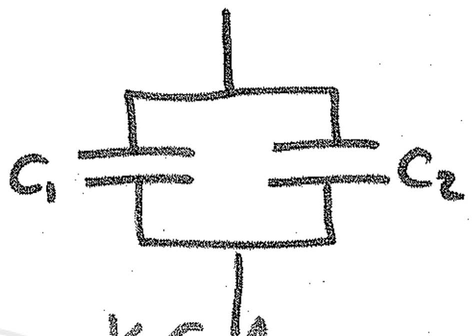
$$\boxed{C_{eq} = \left(\frac{2K_1K_2}{K_1+K_2}\right)C_0}$$

10

**Ex:**




$\Rightarrow$




$$C_1 = \frac{K_1 \epsilon_0}{d}\frac{A}{2}$$

$$C_2 = \frac{K_2 \epsilon_0 A}{2d}$$

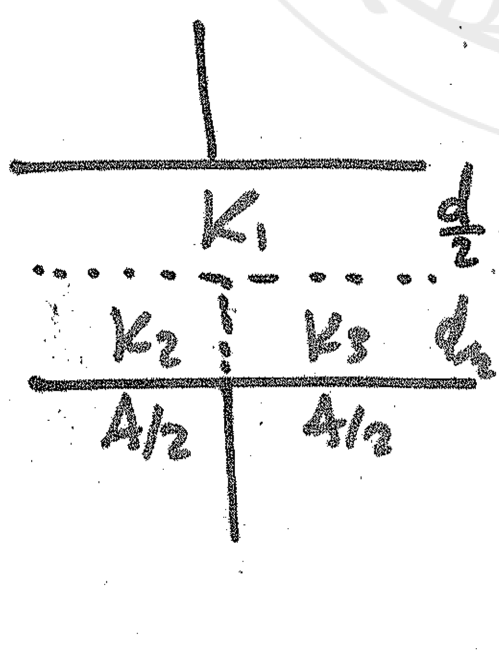
$$C_{eq} = C_1 + C_2$$

$$= \frac{K_1}{2}\frac{\epsilon_0 A}{d} + \frac{K_2}{2}\frac{\epsilon_0 A}{d}$$

$$= \left(\frac{K_1+K_2}{2}\right)\frac{\epsilon_0 A}{d}$$

$$C_{eq} = \frac{K_1+K_2}{2} C_0$$

---

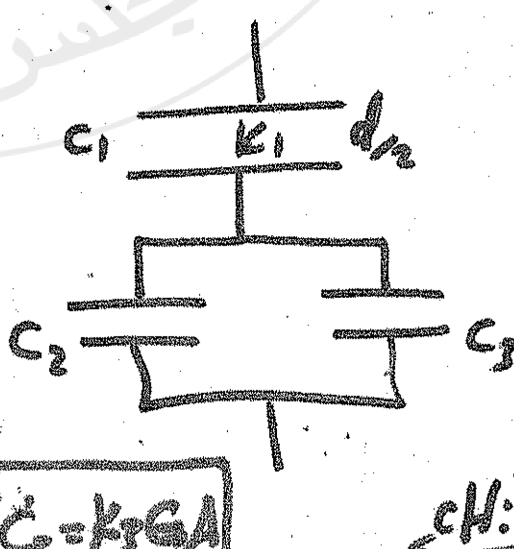



$$C_1 = \frac{K_1 \epsilon_0 A}{d/2}$$

$$C_2 = \frac{K_2 \epsilon_0 \frac{A}{2}}{d/2}$$

$$= \frac{K_2 \epsilon_0 A}{d}$$

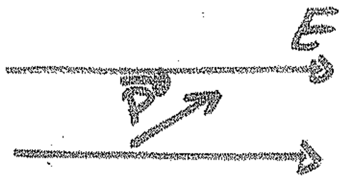
$$C_3 = \frac{K_3 \epsilon_0 A}{d}$$

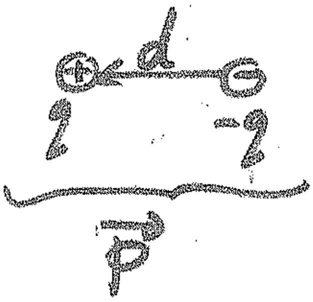



cH: 26

11

**Dipole → inside E**




$\vec{P} = q\vec{d}$

$P = qd$

* $\vec{\tau} = \vec{P} \times \vec{E}$

$\tau = PE\sin\theta_{PE}$

* $U = -\vec{P}\cdot\vec{E}$

$= -PE\cos\theta_{PE}$

$U_{max} = PE \quad (\theta = 180°)$

$U_{min} = -PE \quad (\theta = 0)$

$\Delta U = 2PE$

Q51 $q = \checkmark$ $\quad q = (-1.2\ ,\ 1.1) \quad \vec{E} = (\ )\hat{i} - (\ )\hat{j}$

$-q = (1.4, -1.3)$

a) $\vec{P} = q\vec{d}$

$= q(-2.6\hat{i} + 2.4\hat{j})$

b) $\vec{\tau} = \vec{P} \times \vec{E}$

$\vec{\tau} \perp \vec{P}, \vec{E}$

c) $U = -\vec{P}\cdot\vec{E}$

$= \quad J$

d) $\Delta U = 2PE$

$$I_{av} = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = I \Delta t$$

$$I_{ins} = \frac{dq}{dt} \Rightarrow \Delta q = \int_{t_1}^{t_2} I\,dt$$

Ex: If $q = 4t^2 - 5t + 1$, Find the Current at $t = 2$ sec.

$$\Rightarrow I_{ins} = \frac{dq}{dt} = 8t - 5 \Rightarrow I = 11 \text{ ~~sec~~ Amp.}$$

$$I_{av} = \frac{\Delta q}{\Delta t} = \frac{q_2 - q_1}{t_2 - t_1} \quad t = 0 \rightarrow 2.$$

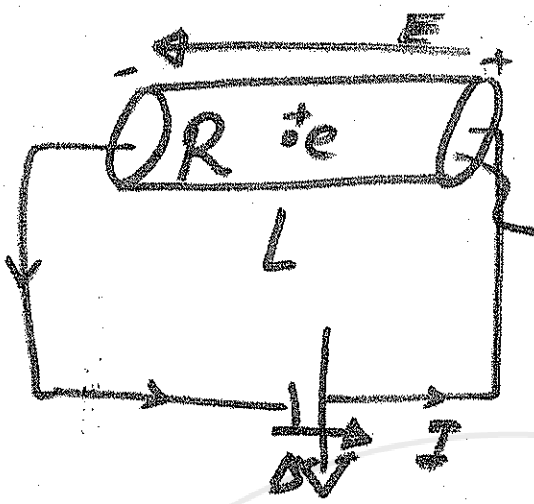
$$q_1(t=0) = 1$$

$$q_2(t=2) = 4(4) - 5(2) + 1 = 7$$

$$\Rightarrow I = \frac{7-1}{2-0} = \frac{6}{2} = 3\text{ A}.$$

Ex: $I = 6t - 2$, Find charge from $t=0$ to $t=1$.

$$\Delta q = \int_0^1 (6t-2)\,dt = 3t^2 - 2t\Big|_0^1 = 3 - 2 = 1 \text{ Col}.$$




$A = \pi r^2$ (r = radius)

$Vol = AL$

$q = I * t$

$$\boxed{\Delta V_R = I * R}$$ — Ohm's law.

$P_R = IV = I^2 R = \frac{V^2}{R}$

$U = P * t$

$I = \frac{q}{t}$

$q = ne$

$I = n' e v_d A$

$v_d = \frac{L}{t}$

$J = \frac{I}{A} = n' e v_d$ — $J = \sigma E$

$R = \frac{\rho L}{A}$

$\rho = \frac{1}{\sigma}$

$\Delta V_R = E * L$

$F = eE$

$\sigma = \frac{n' e^2 \tau}{m_e}$

$q$ = charge inside the resistance.

$I$ = Current

$t$ = time of current moving

$\Delta V$ = Potential (voltage) difference accross the $\underline{R}$.

$n$ = number of electrons pass through the wire.

$e$ = electron charge ($e = +1.6 * 10^{-19} C$)

$n'$ = electron's density ($\underline{no}$ of $\underline{e}$ in Unit volume) $[e/m^3]$

$v_d$ = drift velocity (سرعة الانسياق)

$A$ = Area of the wire. ($m^2$)

$L$ = length of the wire.

$J$ = Current density $[A/m^2]$

$R$ = Resistance (ohm = $\Omega$)

$\rho$ = resistivity ($\Omega \cdot m$)

$\sigma$ = Conductivity $(\Omega \cdot m)^{-1}$

$E$ = Electric field.

$P$ = Power (energy rate) $\Rightarrow$ watt

$U$ = energy in $\underline{R}$.

$\tau$ = av. time interval between two Successive collisions.

$m_e$ = mass of

$\underline{Ex}$: Resistance of $\underbrace{\text{length } 10m}_{(L)}$ and radius of ④

$\underbrace{\left(\frac{2}{\sqrt{\pi}} cm\right)}_{r \rightarrow A = 4*10^{-4} m^2}$, has a Conductivity of $\underbrace{4*10^{6}}_{\sigma} (\Omega \cdot m)^{-1}$,

Connected accross a pot. diff of $\underbrace{20\text{-}V}_{\Delta V}$ for

$\underbrace{10\text{-}sec}_{t}$, Find:

1) resistivity 2) resistance 3) Current

4) charge 5) no of $\underline{\underline{e}}$ Passes through the wire. (n)

6) drift velocity 7) no of $\underline{\underline{e}}$ Per Unit Vol. (n')

8) Current density 9) Electric Field

10) Power 11) energy 12) Force on $\underline{\underline{e}}$.

13) time between $\underline{\underline{2}}$ Successive Collisions ($\tau$)

⑤

$$\underline{\underline{R}} \longrightarrow L$$
$$\quad \longrightarrow A \longrightarrow r$$

$$\begin{tikzcd}
r \arrow[r, "\text{تربيع}"] & A \arrow[r, "\text{مقلوب}"] \arrow[l, bend left=60, "\text{الجذر}"] & L \arrow[r, "\text{تربيع}"] \arrow[l, bend left=60, "\text{مقلوب}"] & R \arrow[l, bend left=60, "\text{الجذر}"]
\end{tikzcd}$$

$$r_2 = 2r_1 \Longrightarrow A_2 = 4A_1 \Longrightarrow L_2 = \frac{1}{4}L_1 \Longrightarrow R_2 = \frac{1}{16}R_1$$

$$A_2 = 2A_1 \longrightarrow L_2 = \frac{1}{2}L_1 \longrightarrow R_2 = \frac{1}{4}R_1$$

$$R_2 = 3R_1 \Longrightarrow L_2 = \sqrt{3}\,L_1 \Longrightarrow A_2 = \frac{1}{\sqrt{3}}A_1 \Longrightarrow r_2 = \frac{1}{3^{1/4}}r_1$$

$$\Downarrow$$

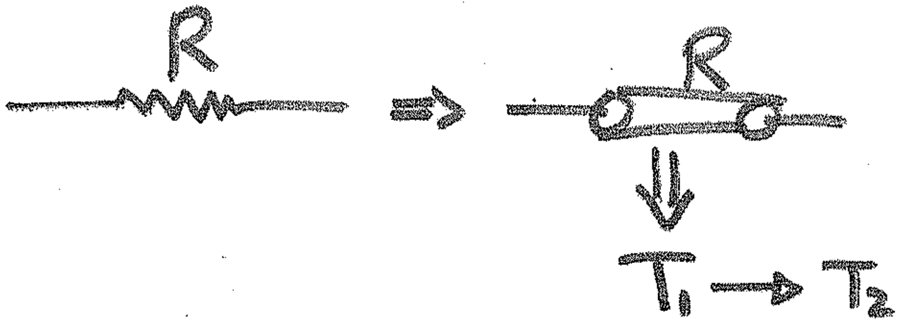
$$\left(\frac{1}{3}\right)^{\frac{1}{4}}$$

$* \; r_2 = 2r_1$

$$A_1 = \pi r_1^2$$
$$A_2 = \pi r_2^2 = \pi(2r_1)^2 = 4\pi r_1^2$$
$$A_2 = 4A_1$$

$$V_1 = V_2$$
$$L_1A_1 = L_2A_2$$
$$L_1\cancel{A_1} = L_2\,4\cancel{A_1}$$
$$L_2 = \frac{1}{4}L_1$$

$$R_1 = \frac{\rho L_1}{A_1}$$
$$R_2 = \frac{\rho L_2}{A_2} = \frac{\rho L_1}{4 * 4A_1}$$
$$= \frac{\rho L_1}{A} * \frac{1}{16} = \underline{R_1}$$




$R_0$ : orginal resistance.

R : new resistance

$\alpha$ : Temp. Coeff. of resistance. $\left(\frac{1}{C^\circ}\right)$

$$R = R_0[1 + \alpha(T_f - T_i)]$$

$$\rho = \rho_0[1 + \alpha(T_f - T_i)]$$

---

<u>Ex</u> : $\alpha = 2 * 10^{-5}/c$

$T_i = 50°c$ $\qquad T_f = ??$

$R_0 = 10\Omega \longrightarrow R = 20\Omega$

$$20 = 10[1 + 2*10^{-5}(T_f - 50)]$$

$$T_f = 50050\,C^\circ.$$

---

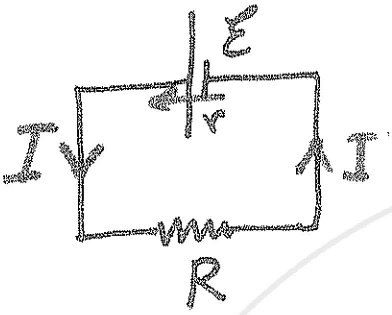
$\frac{\Delta R}{R} \Rightarrow R = R_0 + R_0\alpha(\Delta T)$ | $\frac{\Delta R}{R}$ : fractional change in

$\underline{\Delta R} = \alpha \Delta T$

1

# CH: ~~28~~ 25 Electrical Circuits

الدارات الكهربائية



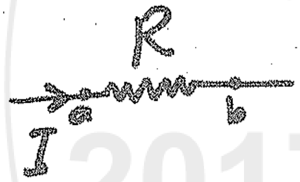

$I$: Current (A)

$R$: external resistance ($\Omega$)

$r$: internal resistance ($\Omega$)

$\mathcal{E}$: electromotive force (V)

Resistance :




$$\Rightarrow \boxed{\underset{(R)}{V_{ab}} = I R} \text{ — ohm's law.}$$

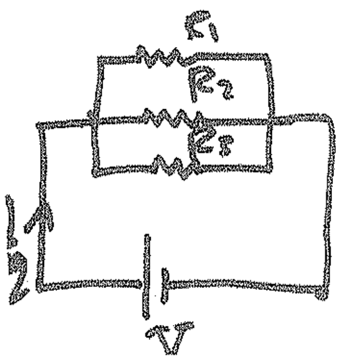
$$P_R = I * V_R = I^2 R = \frac{V^2}{R} .$$

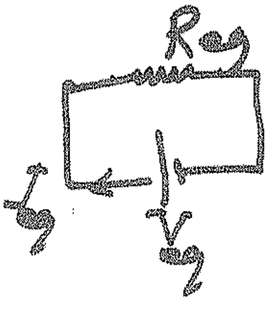
$$U = P * t$$

## Resistors' Connection

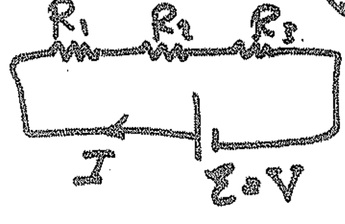
توصيل المقاومات

In Parallel (التوازي)

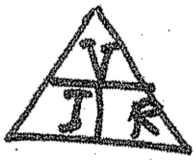



In Series (التوالي)



## التوازي




## التوالي

**التوازي**

التيار يتوزع:

$$I_{eq} = I_1 + I_2 + I_3 + \dots$$

الجهد متساوي:

$$\boxed{V_{eq} = V_1 = V_2 = V_3 = \dots}$$

المقاومة مقلوبة:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

**التوالي**

التيار متساوي

$$\boxed{I_{eq} = I_1 = I_2 = I_3 = \dots}$$

الجهد يتوزع

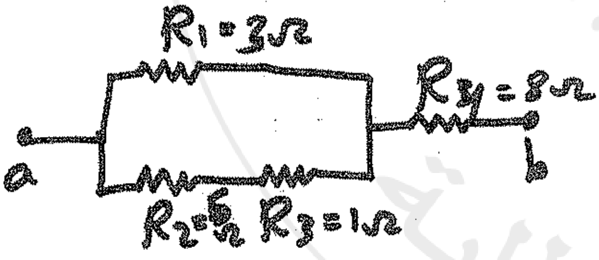
$$V_{\text{بطارية}} = V_{eq} = V_1 + V_2 + V_3 + \dots$$

المقاومة مجموعة

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

---

**Ex**: Find the equivalent resistance ($R_{eq}$):




$R_2, R_3 \xrightarrow{\text{توالي}} R_5$

$R_1, R_5 \xrightarrow{\text{توازي}} R_6$

$R_4, R_6 \xrightarrow{\text{توالي}} R_7 = R_{eq}$.

$$* \; R_5 = R_2 + R_3 = 5 + 1 = 6\,\Omega$$

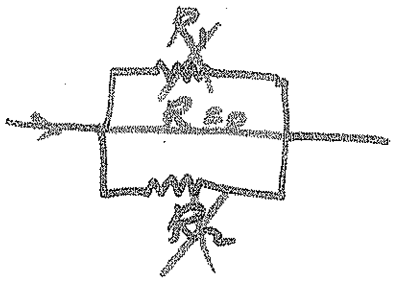
$$\frac{1}{R_6} = \frac{1}{R_5} + \frac{1}{R_1}$$

$$\frac{1}{R_6} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

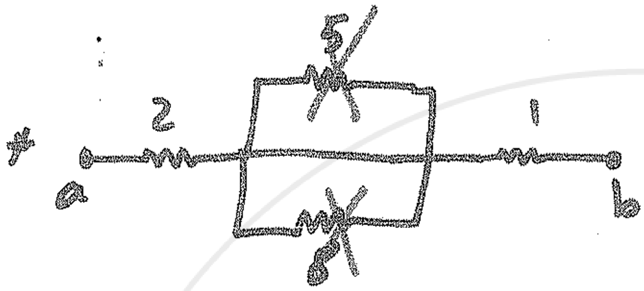
$$\boxed{R_6 = 2\,\Omega}$$

$$R_7 = R_4 + R_6 = 8 + 2$$

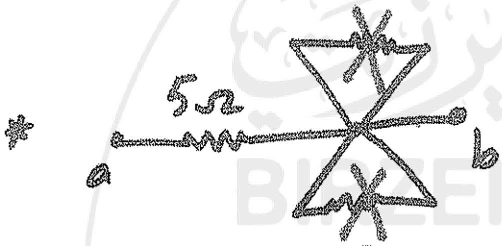
$$\boxed{R_7 = 10\,\Omega}$$



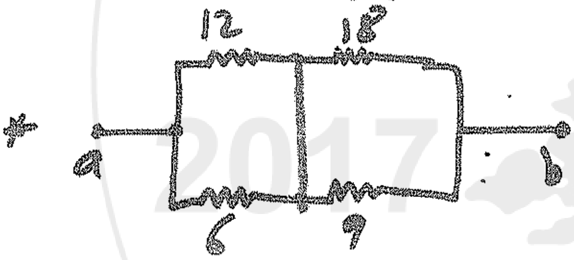

$\Rightarrow R_g = 0$



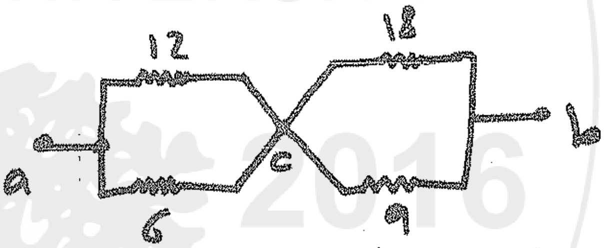

$R_{ab} = 3\Omega$.



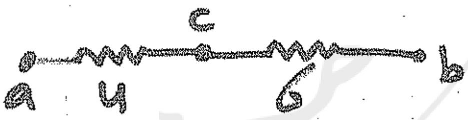

$R_{ab} = 5\Omega$




$\Rightarrow$

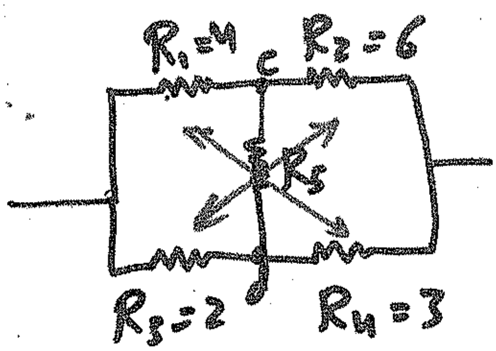


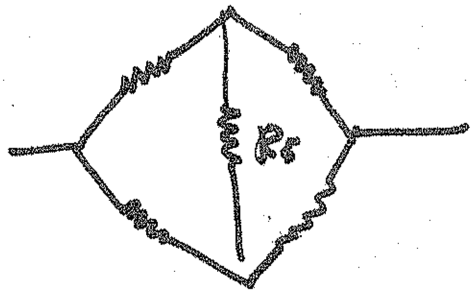

توازي $\Downarrow$ $\Downarrow$ توازي



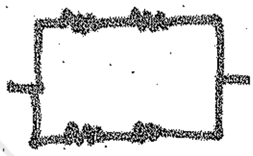

$\Downarrow$

$R_{ab} = 10\Omega$.




$$\underbrace{R_1 * R_4 \overset{?}{=} R_2 * R_3}_{\Downarrow} \Longrightarrow R_5 \text{ تلغى} \Longrightarrow$$



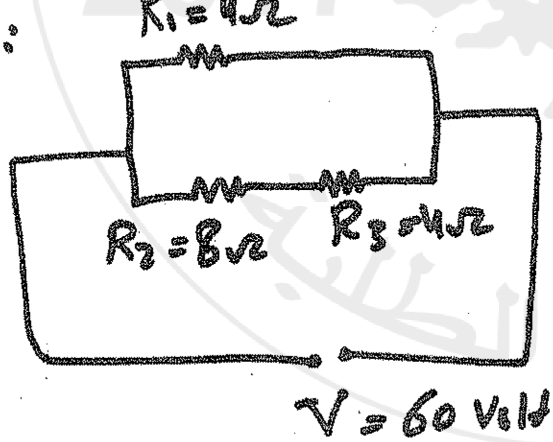
$$-Ⓖ- = I_{R_5} = 0$$

$$\Downarrow$$

$$\boxed{V_c = V_d} \quad \text{or} \quad \boxed{V_{cd} = 0}$$

---

**Ex:**




Find : 1) Req.

2) Voltage and Curren[t] in each resistor.

3) Power in $R_3$

١) نرسم خريطة مع تعيين
النقط الاضافي.
٢) نجد Req
٣) نجد جميع المعلومات عن العنصر الاضافي
٤) نجد مقبة (هي) [illegible]

$$R_2, R_3 \xrightarrow{\text{توالي}} R_4$$

$$R_1, R_4 \xrightarrow{\text{توازي}} R_5 = R_{eq}$$

$$V_5 = 60 \text{ Volt}$$

(5)

(1)

$R_4 = R_2 + R_3$

$= 8 + 4 = 12\,\Omega$

$\frac{1}{R_5} = \frac{1}{R_1} + \frac{1}{R_4}$

$= \frac{1}{4} + \frac{1}{12}$

$\boxed{R_5 = 3\,\Omega}$

(2)

$V_5 = 60\text{ V}$

$R_5 = 3\,\Omega$

$\Rightarrow I_5 = \frac{V_5}{R_5} = \frac{60}{3} = 20\text{A}.$

$V_5 = V_4 = V_1 = 60\text{ V}$

$\rightarrow I_1 = \frac{V_1}{R_1} = \frac{60}{4} = 15\text{A}$

$\rightarrow I_4 = \frac{V_4}{R_4} = \frac{60}{12} = 5\text{A}$

$I_4 = I_2 = I_3 = 5\text{A}$

$\rightarrow V_3 = I_3 R_3 = 5 * 4 = 20\text{ Volt}$

$\rightarrow V_2 = I_2 R_2 = 5 * 8 = 40\text{ Volt}.$

(3)

$P_3 = I_3^2 R_3$

$= (5)^2 * 4$

$= 25 * 4$

$= 100\text{ watt}$

$P_1 = I_1^2 R_1$

$= (15)^2 * 4$

$= 900\text{ watt}$

$P_2 = I_2^2 R_2$

$= (5)^2 * 8$

$= 200\text{ watt}$

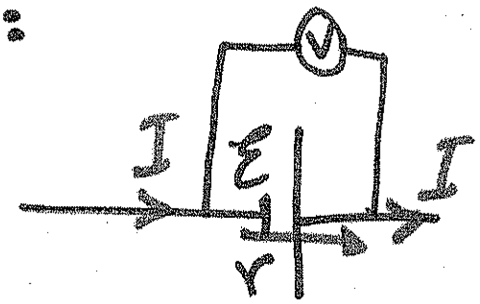
$P_5 = I_5^2 R_5$

$= (20)^2 * 3$

$= 400 * 3$

$= 1200\text{ watt}$

# Battery :




* $V_\varepsilon$ : Potential difference accross the battery.

$$\boxed{V_\varepsilon = \varepsilon \underset{\text{I against } \varepsilon}{\overset{\text{I with } \varepsilon}{\mp}} Ir}$$

$V_\varepsilon = \varepsilon$ if

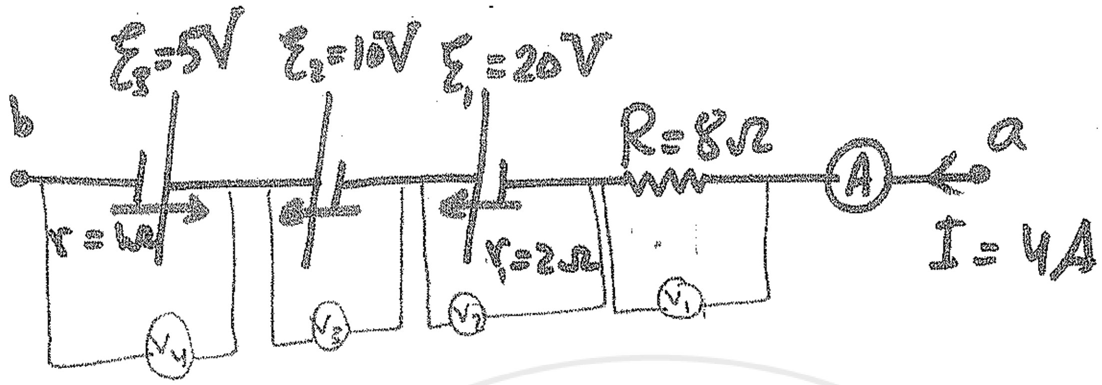
| | |
|---|---|
| $r=0$ | ideal battery |
| $I=0$ | open circuit |


$\Rightarrow \ominus$ discharging


$\Rightarrow \oplus$ charging

* Power of battery : $\boxed{P_\varepsilon = I\varepsilon}$

* energy $(U) \Rightarrow U = P * t$ .




What are the readings of A and V's.



## ملخص للقوانين

$V = \boxed{V}$

$\boxed{\mathcal{E}}$

$V_{\mathcal{E}} = \mathcal{E} \mp IR$

$\boxed{R}$

$V = IR$

(watt) = Power [$U = P \times t$]

$\boxed{\mathcal{E}}$

$P = I\mathcal{E}$

$\boxed{R}$

$P_R = I^2 R = \frac{V^2}{R} = IV$

$A = I = 4A.$

$V_1 = V_R = IR = 4 * 8 = 32$ Volt.

$V_2 = V_{\mathcal{E}_1} = \mathcal{E}_1 - Ir_1$

$= 20 - 4 * 2$

$= 12$ Volt.

$V_3 = V_{\mathcal{E}_2} = \mathcal{E} = 10$ Volt.

$V_4 = V_{\mathcal{E}_3} = \mathcal{E}_3 + Ir_3$

$= 5 + 4 * 1$

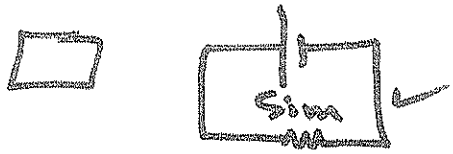
$= 9$ Volt.

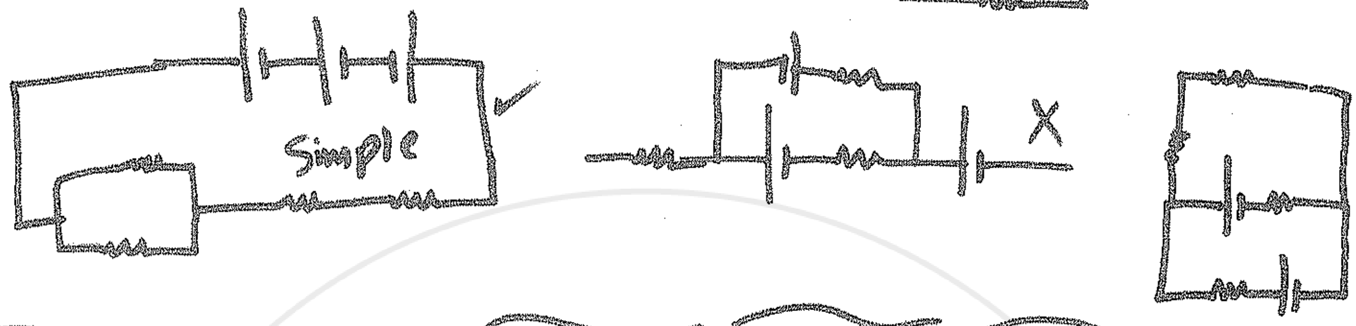
---

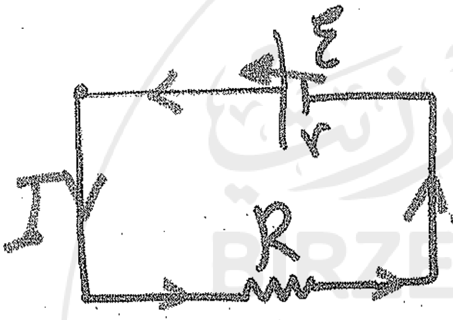

$I_a = I_b = I_c = I_d = I_e = I_f = I_g = I_h$

$V_a = V_b > V_c = V_d < V_e = V_f > V_g = V_h$

Simple circuits:



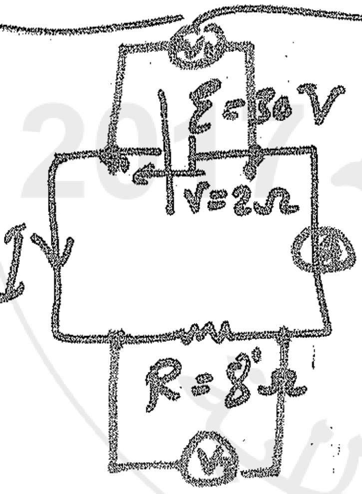




$$\Rightarrow \boxed{\sum \mathcal{E} = I \sum R}$$

R or r

Ex:




Find: 1) Reading of Ⓐ.

2) " " $V_1$, $V_2$

3) Power disappated in R

4) " " " r.

5) Produced Power by $\mathcal{E}$

6) Consumed energy in R through 2-min.

1) $\sum \mathcal{E} = I \sum R$

$30 = I(2+8)$

$I = 3A =$ Ⓐ

2) $V_{\mathcal{E}} = \mathcal{E} - Ir$

$= 30 - 3 * 2$

$= 24$ Volt

$V_2 = V_R = IR = 3 * 8$

$= 24$ Volt.

(3) $P_R = I^2 R = (3)^2 * 8 = 72$ watt.

(4) $P_r = I^2 r = (3)^2 * 2 = 18$ watt

(5) $P_\mathcal{E} = I\mathcal{E} = 3 * 30 = 90$ watt.

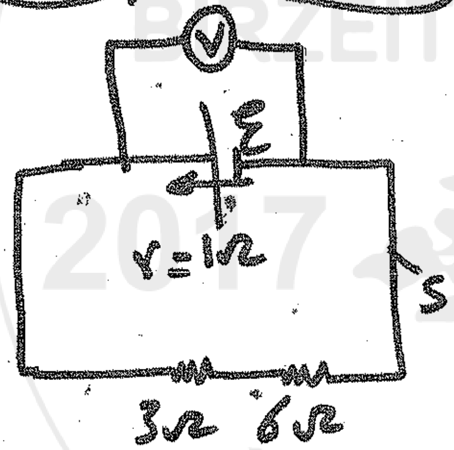
(6) $U_R = P_R * t = 72 * 120$

$= 8640$ J.

(7) drop in $\mathcal{E}$ Potential : $\Rightarrow Ir = 3 * 2 = 6$ Volt

---

Ex :




reading of (V) is 40V when S is open. what is the reading if the S is closed.

S is open:- $I = 0 \Rightarrow$ (V) $= V_\mathcal{E} = \mathcal{E} - \cancel{Ir}$

$\boxed{40 = \mathcal{E}}$

S is closed: (V) $= V_\mathcal{E} = \mathcal{E} - Ir$

$= 40 - 4 * 1$

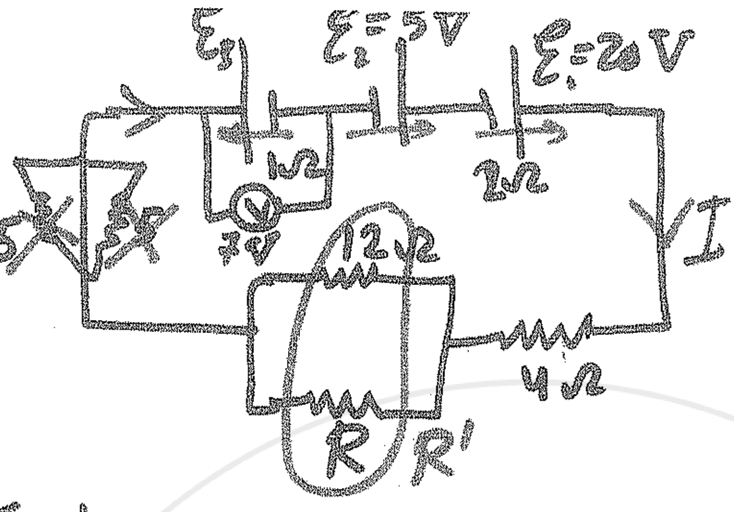
$= 36$ Volt.

$\Sigma \mathcal{E} = I \Sigma R$

$40 = I(1 + 3 + 6)$

$I = 4A$.

Ex :




(11)

* Power in $R = 4\Omega$ is 16 watt.

Find ① $I$ ② $\varepsilon_3$ ③ $R$

$\varepsilon$ x
$R$ x
$r$ ✓
$I$ x

$P_R = I^2 R$

$16 = I^2 * 4$

$I^2 = 4$

$\boxed{I = 2A}$

$V_{\varepsilon_3} = \varepsilon_3 + Ir$

$7 = \varepsilon_3 + 2*1$

$\boxed{\varepsilon_3 = 5 \text{ Volt}}$

$\Sigma \varepsilon = I \Sigma R$

$20 + 5 - 5 = 2(2 + 1 + R' + 4)$

$20 = 2(7 + R')$

$10 = 7 + R'$

$\boxed{R' = 3\Omega}$

$\frac{1}{R'} = \frac{1}{R} + \frac{1}{12}$

$\frac{1}{3} - \frac{1}{12} = \frac{1}{R}$

$\boxed{R = 4\Omega}$

Find current in $R$.
$R$ . 12 توازي $R'$

$\Rightarrow V_{R'} = IR'$
$= 2*3$

$V_R = 6$ Volt
$\Rightarrow I' = \frac{V_R}{R} = \frac{6}{4} = 1.5$

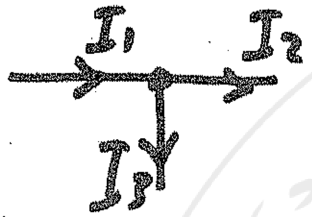
# الدارات المعقدة (الزمن حلقة)

## قانونا كيرتشوف

خطوات حل الدارات المعقدة :-

① نحدد اتجاه التيارات إذا لم تكن محددة (بشكل عشوائي)

② نطبق في قانون كيرتشوف الأول:




$$\sum I_{in} = \sum I_{out}$$

* نستفيد من المعطيات الإضافية إن وجدت.

$$V_{ab} \quad , \quad V \quad , \quad P_{\varepsilon} \quad , \quad P_R \; .$$

③ نحدد مساراً مغلقاً لنا نتحرك عليه.

④ نطبق في قانون كيرتشوف الثاني:

$$(V_{ab} = V_a - V_b) \qquad V_{ab} = \sum IR - \sum \varepsilon$$

معنا: +

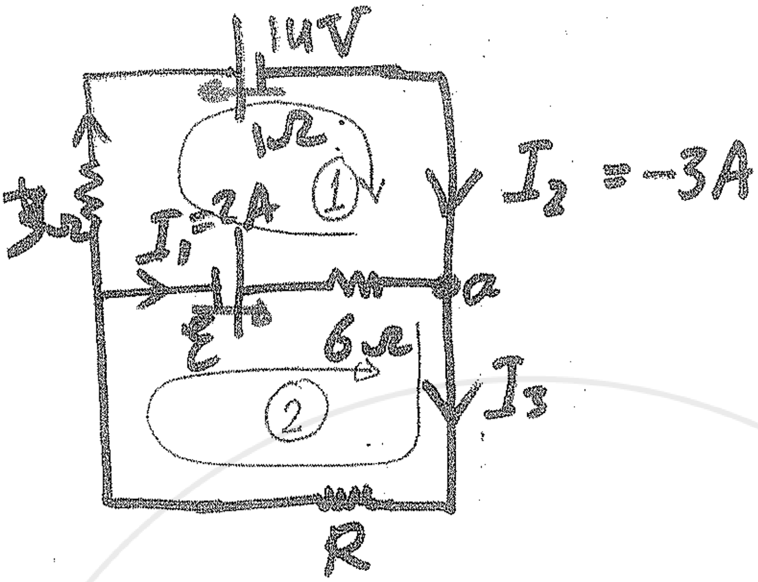
عكسنا: −

* إذا وجدت $I$ سالبة فإن اتجاه $I$ هو عكس الاتجاه المفروض.

* $0 = V_{bb} = V_{aa}$

* إذا تغير أي شيء في الدارة فنحن أمام مسألة جديدة.

Ex:




Find:
1) $I_3$
2) $\varepsilon$
3) $R$

$\sum I_{in} = \sum I_{out}$

$I_1 + I_2 = I_3$

$2 + -3 = I_3$

$\boxed{I_3 = -1A}$

$V_{aa} = (-6*2 + 3*I_2 + 1*I_2) - (-\varepsilon - 14)$

$0 = -12 + 4*(-3) + \varepsilon + 14$

$-\varepsilon = -12 - 12 + 14$

$-\varepsilon = -10 \Rightarrow \boxed{\varepsilon = 10 \text{ Volt}}$

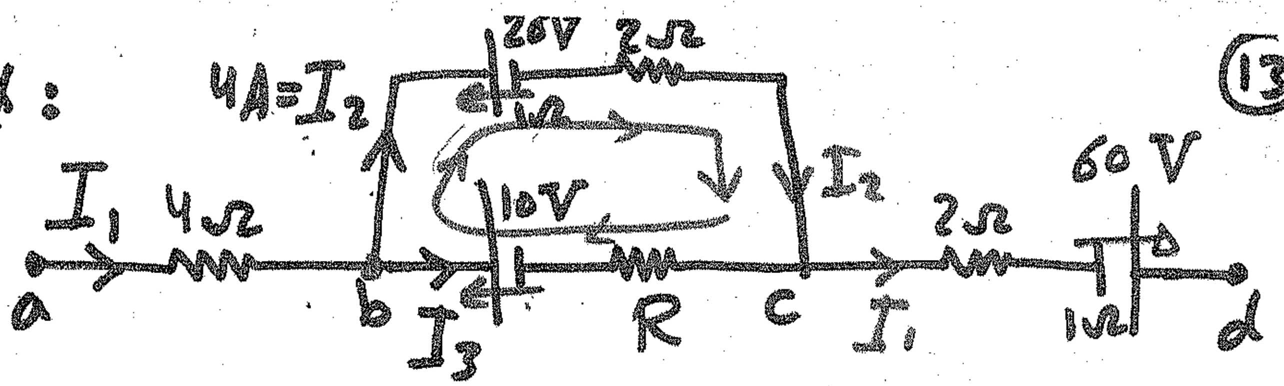
$V_{aa} = (R*I_3 + 6I_1) - (\varepsilon)$

$0 = -R + 6*2 - 10$

$R = 12 - 10$

$\boxed{R = 2\Omega}$

**Ex:** (13) b




If $\underrightarrow{V_{ab}} = 40$ V, Find: 1) $I_3$, 2) R

3) $V_{dc}$

**Sol:** 1

$\Sigma I_{in} = \Sigma I_{out}$

$I_1 = I_2 + I_3$

$\boxed{I_1 = 4 + I_3}$

$V_{ab} = (\Sigma IR) - (\Sigma \mathcal{E})$

$40 = (4I_1) - (0)$

$\boxed{I_1 = 10\,A} \Longrightarrow \boxed{I_3 = 6A}$

3

$\underrightarrow{V_{dc}} = (-3\times10) - (-60)$

$= -30 + 60$

$= 30$ Volt.

2

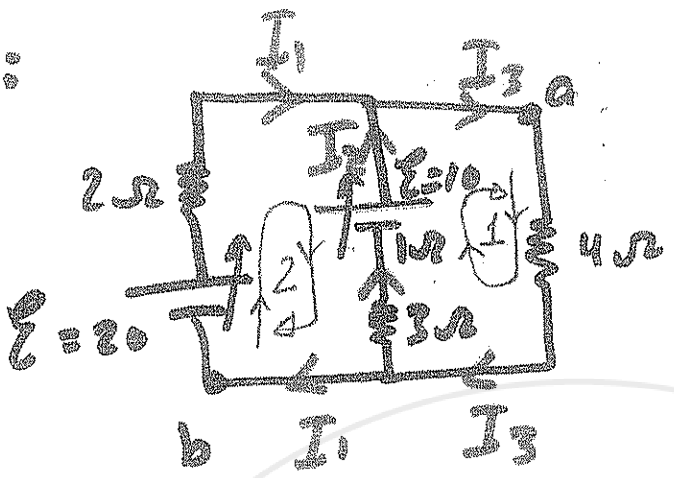
$V_{cc} = (-6*R + 3*4) - (10 - 20)$

$0 = -6R + 12 - 10 + 20$

$6R = 22$

$\boxed{R = 3.7\,\Omega}$

Ex :




Find:

1) each current.
2) $V_{ab}$
3) if $V_b = 10V$, find $V_a$.

$$\sum I_{in} = \sum I_{out}$$

$$\boxed{I_1 + I_2 = I_3}$$

$$V_{aa} = (4I_3 + 4I_2) - (10)$$

$$0 = 4I_3 + 4I_2 - 10$$

$$10 = 4(I_1 + I_2) + 4I_2$$

$$\boxed{10 = 4I_1 + 8I_2} \text{ --- } 1$$

$$V_{bb} = (2I_1 - 4I_2) - (20 - 10)$$

$$0 = 2I_1 - 4I_2 - 10$$

$$\boxed{10 = 2I_1 - 4I_2} \text{ --- } 2$$

$$\Rightarrow I_1 = 3.75\,A$$

$$\boxed{I_2 = -0.625}$$

$$\Rightarrow I_3 = 3.75 + (-0.625)$$

$$\boxed{I_3 = 3.125\,A}$$

② $V_{ab} = (4 * I_3 \quad ) - ( \quad 0 \quad )$ ⑮

$= 4 * 3.125$

$= 12.5$ Volt.

$V_{ab} = (2 * -3.75 \quad ) - (-20 \quad )$

$= -7.5 + 20$

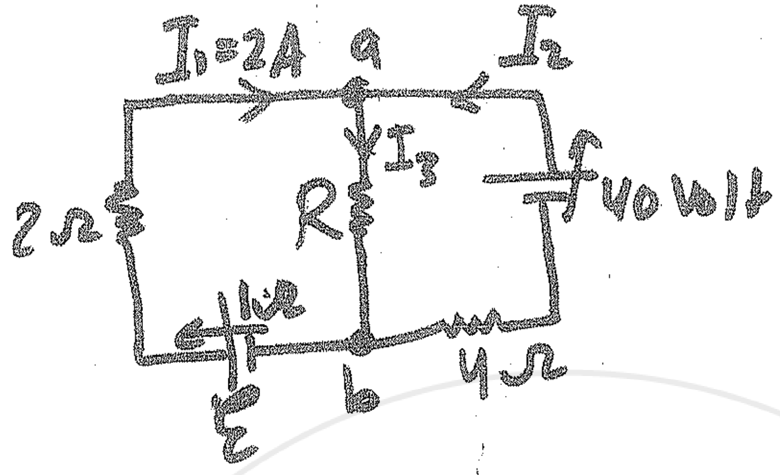
$= 12.5$ Volt.

③ $V_{ab} = V_a - V_b$

$12.5 = V_a - 10$

$\boxed{V_a = 22.5 \text{ Volt}}$

**Ex:**




Find
1) all I's.
2) R
3) $\mathcal{E}$

where $V_{ab} = 24V$.

**Sol**

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_2 = I_3$$

$$\boxed{2 + I_2 = I_3}$$

$$V_{ab} = (-4 * I_2 \quad ) - (-40 \quad )$$

$$24 = -4 I_2 + 40$$

$$-16 = -4 I_2$$

$$\boxed{I_2 = 4A} \Rightarrow \boxed{I_3 = 6A}$$

---

$$V_{ab} = (R * 6 \quad ) - ( \quad 0 \quad )$$

$$24 = 6R$$

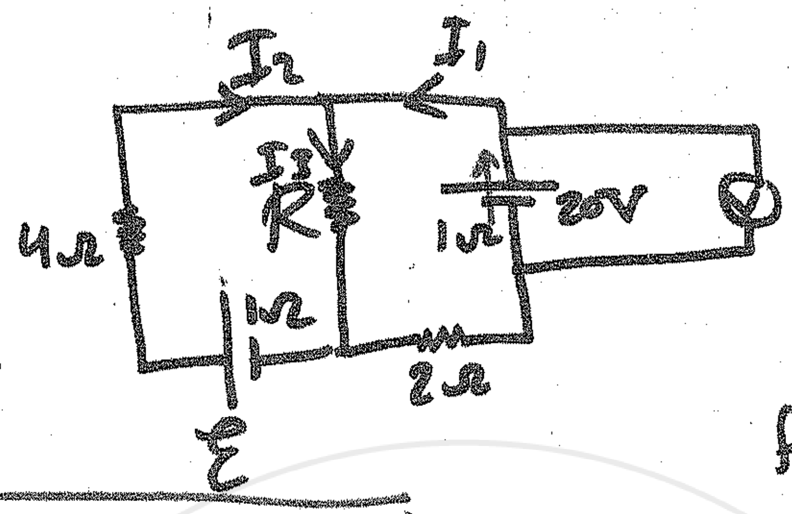
$$\boxed{R = 4\Omega}$$

$$V_{ab} = (-3 * 2 \quad ) - (-\mathcal{E} \quad )$$

$$24 = -6 + \mathcal{E}$$

$$\boxed{\mathcal{E} = 30\,V}$$

Ex :




V = 14V

$P_{4\Omega} = 64$ watt

find : I's , R, ε

$$\boxed{I_1 + I_2 = I_3}$$

$P_{4\Omega} = I_2^2 R$

$64 = I_2^2 \, 4$

$I_2^2 = 16$

$\boxed{I_2 = 4A}$

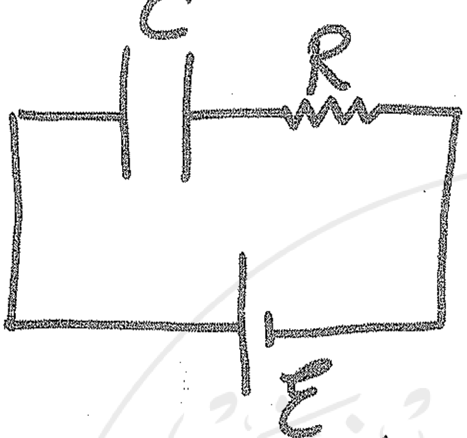
$V_{\varepsilon} = \varepsilon - I_1 r$

$14 = 20 - I_1 * 1$

$\boxed{I_1 = 6A} \Longrightarrow \boxed{I_3 = 10A}$

# RC - circuit

دارة مقاومة ومكثف




$$\mathcal{E} = V_R + V_C$$

$$\mathcal{E} = IR + \frac{q}{C}$$

| | $t=0$ | long time $t\to\infty$ |
|---|---|---|
| $I$ | $I_{max}$ | $0$ |
| $q$ | $0$ | $q_{max}$ |

$$I_{max} = \frac{\mathcal{E}}{R}$$

$$q_{max} = C\mathcal{E}$$

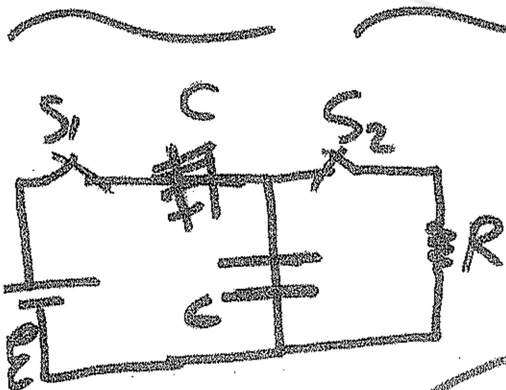
$\tau$ : Time Constant

ثابت الزمن السعوي

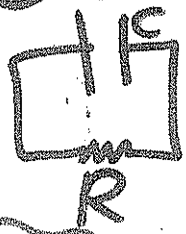
$$\tau = RC$$

$$I = I_{max}\, e^{-t/RC}$$

$$q = q_{max}\left(1 - e^{-t/RC}\right)$$



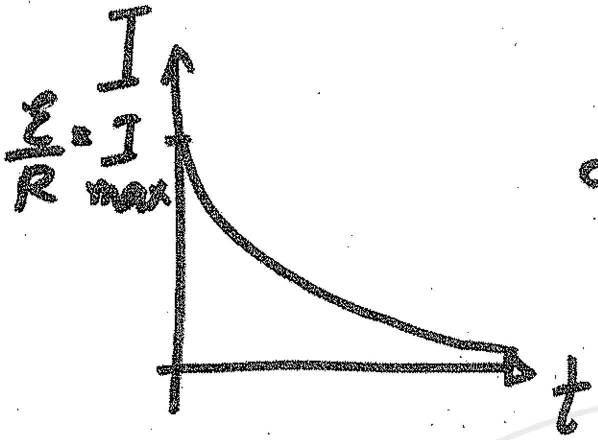

## discharging

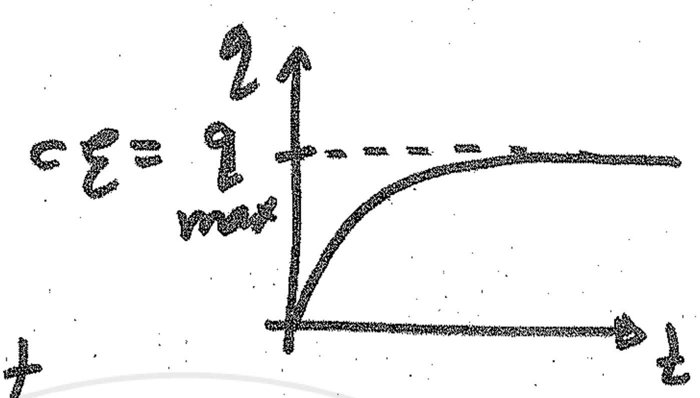


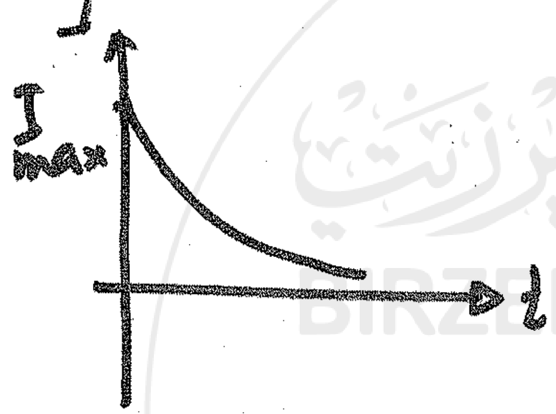

| | $t=0$ | $t=\infty$ |
|---|---|---|
| $I$ | $I_{max}$ | $0$ |
| $q$ | $q_{max}$ | $0$ |

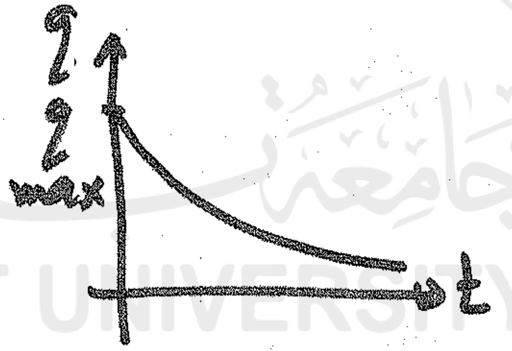
$$I = I_{max}\, e^{-t/RC}$$

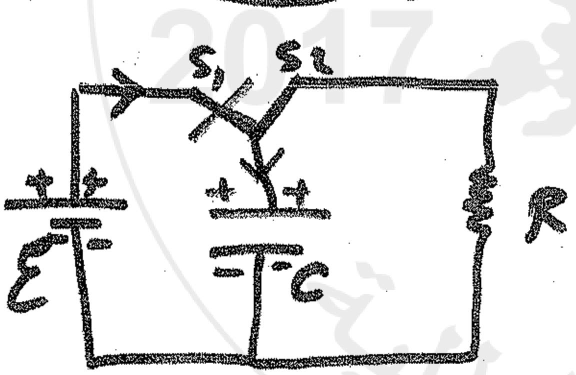
$$q = q_{max}\, e^{-t/RC}$$


I
$\frac{\mathcal{E}}{R} = I_{max}$
t

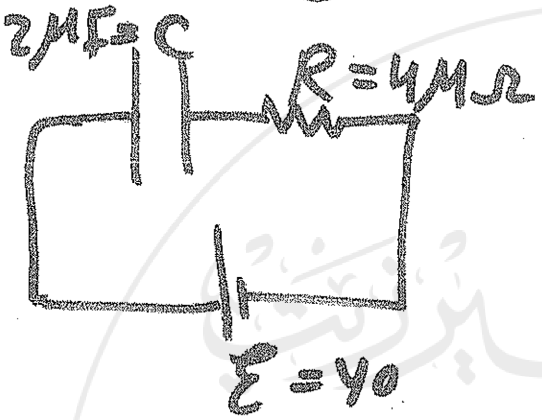

q
$C\mathcal{E} = q_{max}$
t


I
$I_{max}$
t


q
$q_{max}$
t


$S_1$
$S_2$
$\mathcal{E}$
C
R

Ex : RC circuit of $R = 4M\Omega$ and $C = 2\mu F$, R and C were Connected to a battery $\mathcal{E} = 40$ Volt as figure: (20)




Find:

1) max. charge and current.
2) Time Constant ($\tau$)
3) charge and Current at $t = 4$ sec.
4) Voltage of Capacitor and V. of resistor at $t = 4$ sec.
5) charge and current after 4-time Const.
6) time needed to reach to half of max. Current (or half of resistor voltage)
7) time needed to reach to 30% of max charge.
8) energy in Capacitor after 4 sec

$C = 2\mu F \quad R = 4M\Omega \quad \mathcal{E} = 40\,\text{Volt}.$ (21)

1) $q_{max} = C\mathcal{E} = 2\mu * 40 = 80\mu C.$

$I_{max} = \dfrac{\mathcal{E}}{R} = \dfrac{40}{4*10^{6}} = 10\mu A.$

2) $\tau = RC = 4*10^{6} * 2*10^{-6} = 8\text{-sec}.$

3) $q = q_{max}(1 - e^{-t/RC})$

$= 80*10^{-6}(1 - e^{-4/8})$

$= 31.5*10^{-6}\ C$

$I = I_{max}\, e^{-t/RC}$

$= 10*10^{-6} * e^{-4/8}$

$= 6.1*10^{-6} A$

4) $V_c = \dfrac{q}{C} = \dfrac{31.5\mu}{2\mu} = 15.75\ V.$

$V_R = IR = 6.1*10^{-6} * 4*10^{6} = 24.4\ \text{Volt}$

$V_R = \mathcal{E} - V_c = 40 - 15.75 = 24.25\ \text{Volt}.$

5) $q = 80*10^{-6}[1 - e^{-4\tau/\tau}]$

$= 78.5*10^{-6} C.$

$I = 10*10^{-6}\, e^{-4\tau/\tau}$

$= 0.18\mu A.$

6) $I = I_{max}\, e^{-t/\tau}$

$\frac{1}{2}\cancel{I_{max}} = \cancel{I_{max}}\, e^{-t/\tau}$

$\frac{1}{2} = e^{-t/\tau} \longrightarrow \ln 0.5 = \cancel{\ln}\, \cancel{e}^{-t/8}$

$\cancel{-}0.69 = \cancel{-}\frac{t}{8}$

$\boxed{t = 5.52\,sec}$



7) $q = q_{max}\left(1 - e^{-t/\tau}\right)$

$\frac{30}{100}\cancel{q_{max}} = \cancel{q_{max}}\left(1 - e^{-t/8}\right)$

$0.3 = 1 - e^{-t/8}$

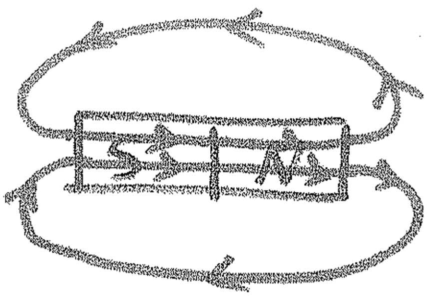
$\cancel{-}0.7 = \cancel{-}e^{-t/8}$ ($\underline{\ln}$)

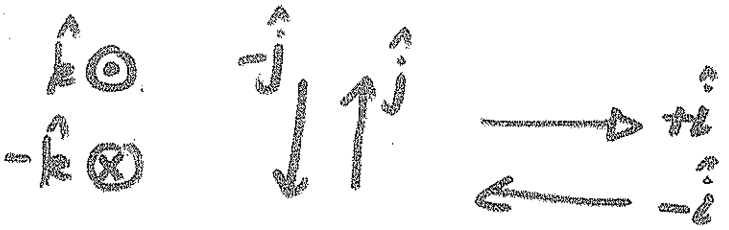
$\ln 0.7 = \cancel{\ln}\, \cancel{e}^{-t/8}$

$-0.36 = -\frac{t}{8}$

$\boxed{t = 2.88\,sec}$

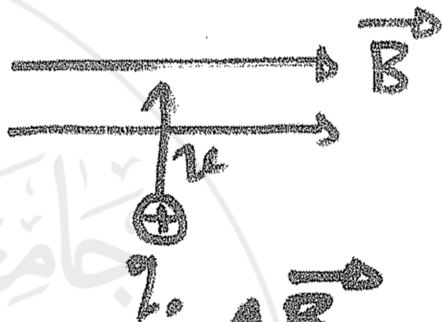
CH: ~~29~~ 26 Magnetic field ($\vec{B}$) ①



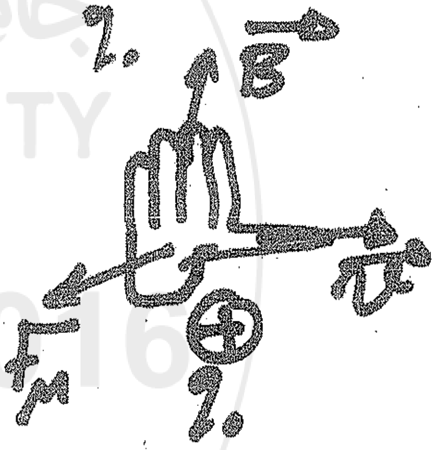

$$\vec{F}_M = q\,\vec{v}\times\vec{B}$$

$$F = q\,v\,B\sin\theta_{vB}$$




$F = 0 \rightarrow v = 0$ (rest)

$\quad\;\; \rightarrow \theta = 0^\circ, 180^\circ$





$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \| \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad |\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta_{AB}$$

$$= (AB - AB_x)\,\hat{i} - (AB - AB)\,\hat{j} + (AB - AB)\,\hat{k}$$

**Ex**: If $\vec{v}$ is given by:

$$\vec{v} = 2\hat{i} - 3\hat{j} + \hat{k}$$

and $\vec{B} = \hat{i} + 2\hat{j} + 2\hat{k}$ . $q_o = 2 \times 10^{-4} C$

Find: 1) Force as a vector.

2) magnitude of the force.

1)

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= (-6 - 2)\hat{i} - (4 - 1)\hat{j} + (4 + 3)\hat{k}$$

$$\vec{v} \times \vec{B} = -8\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\vec{F}_M = q_o \vec{v} \times \vec{B} = 2 \times 10^{-4} [-8\hat{i} - 3\hat{j} + 7\hat{k}]$$

$$= (-16\hat{i} - 6\hat{j} + 14\hat{k}) \times 10^{-4} \text{ New.}$$

2) $|\vec{F}_M| = \sqrt{16^2 + 6^2 + 14^2} \times 10^{-4} = 22 \times 10^{-4} N.$

* If: find $\theta_{vB} \Rightarrow |\vec{F}| = q_o v B \sin\theta_{vB}$

$$|\vec{v}| = v = \sqrt{2^2+3^2+1^2} = \sqrt{14}$$

$$|\vec{B}| = B = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$$

$$\Rightarrow 22 \times \cancel{10^{-4}} = 2 \times \cancel{10^{-4}} \times \sqrt{14} \times 3 \sin\theta_{vB}$$

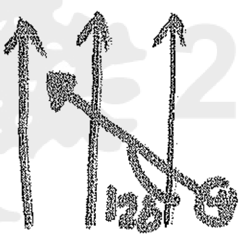
$$\sin\theta = 0.98 \Rightarrow \boxed{\theta = 78.5}$$



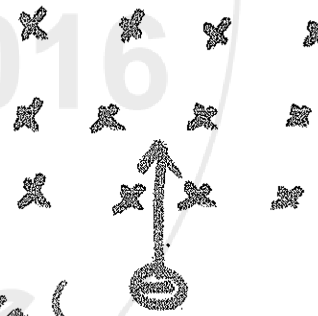
**Ex**: In the figures, Find $\vec{F}_M$: $|q| = 3\times10^{-6}$ C, $v = 10$ m/s, $B = 2$ Tesla


$F_M = 60\times10^{-6}\sin 90$
$= 60\times10^{-6}$ N

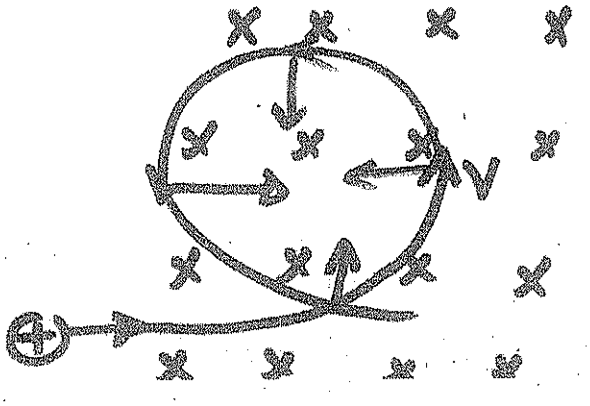
$F_M = 0$




$F_M = 60\times10^{-6}\sin 60$
$= 30\sqrt{3}\times10^{-6}$ N
$(-\hat{k})$



$F_M = 3\times10^{-6}\times10\times2$
$\sin 90$
$= 60\times10^{-6}$ N
$(+\hat{i})$




Circular motion.
$F_m$ toward Center

$$F_c = \frac{mv^2}{r} \qquad a_c = \frac{v^2}{r}$$

$a_c$ → centripetal accel.
التسارع المركزي

$$qvB = \frac{mv^2}{r}$$

$$\boxed{r = \frac{mv}{qB}}$$

$$v = \frac{2\pi r}{T_p}$$ $T_p$ : Period الزمن الدوري

$$T_p = \frac{2\pi r}{v} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

$f \equiv$ frequency التردد $\rightarrow f = \frac{1}{T_p} = \frac{v}{2\pi r} = \frac{qB}{2\pi m}$

* $\omega \equiv$ angular freq. $\Rightarrow \omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}$

Ex: If $q = 4\mu C$ entered a Uniform mag. field $B = 4\,T$ with Const speed $v = 10\,m/s$ and if $m = 2\times10^{-5}\,kg$, Find: (5)

1) magnetic force.
2) centripetal force

$$F_m = F_c = \begin{cases} qVB\sin\theta \\ \frac{mv^2}{r} \end{cases}$$

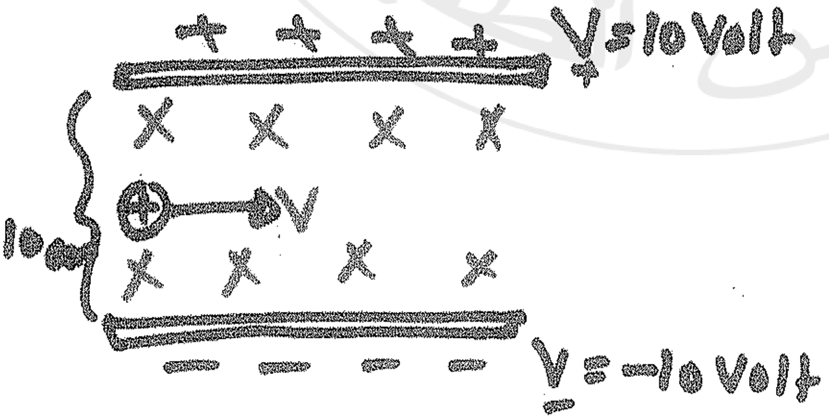
3) accel. $\Rightarrow a_c = \frac{F_c}{m}$

4) radius. $\Rightarrow r = \frac{mv}{qB}$

5) Period $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

6) freq. $\longrightarrow f = \frac{1}{T}$

7) ang. freq. (velocity) $\rightarrow \omega = 2\pi f = \frac{2\pi}{T}$

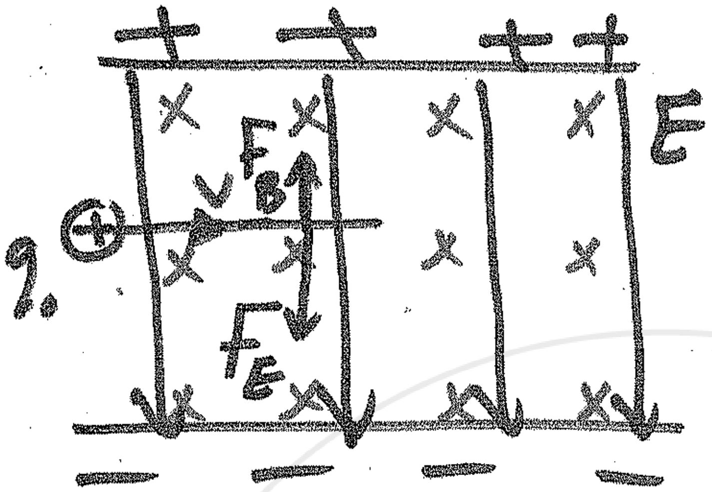



$B = 20\,T$

$q_0 = 4\times10^{-6}\,C$

$v = 20\,m/s$

① $E$
② $F_E$
③ $F_B = F_m$
④ Lorentz force
⑤ $V$ to ensure $q$ moves in a straight line




① $\Delta V = Ed$

$(10 - ^-10) = E \; 10\times10^{-2}$

$E = \dfrac{20}{10\times10^{-2}} = 200 \; N/C$

② $F_E = qE = 4\times10^{-6} * 200 = 800*10^{-6} N.$ $(-\hat{j})$

③ $F_B = qvB\sin\theta = 4\times10^{-6} * 20 * 20 \sin 90$
$= 1600*10^{-6} N$
$(+\hat{j})$

④ $\vec{F}_{lorentz} = \vec{F}_E + \vec{F}_B$

$F_{lor} = F_B - F_E = 800*10^{-6}(+\hat{j})$

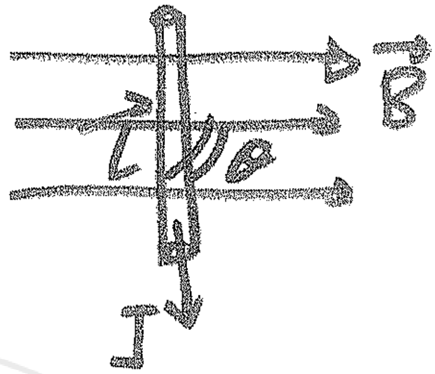
⑤ $F_E = F_B$

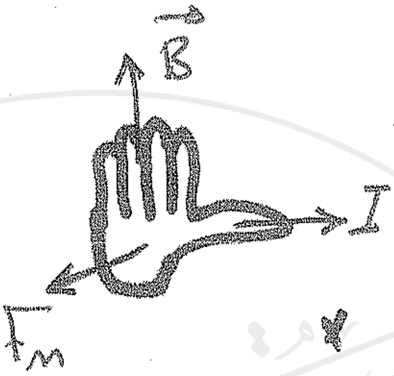
$\cancel{q}E = \cancel{q}vB$

$\boxed{V = \dfrac{E}{B}} \Rightarrow V = \dfrac{200}{20} = 10 m/s.$

$\vec{F}_B = I \vec{l} \times \vec{B}$

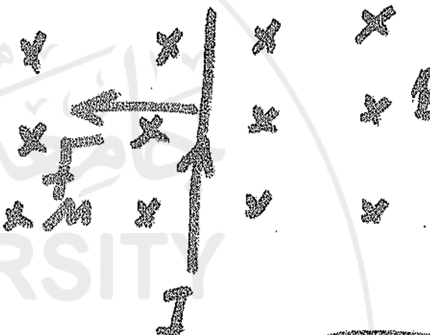
$F_B = I l B \sin\theta_{lB}$



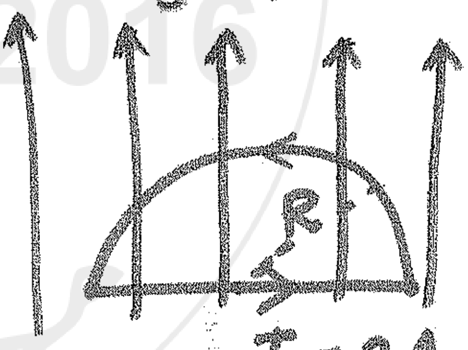

$F = 0$

$\rightarrow \theta = 0^\circ, 180^\circ$




Ex: Find:

1) Net force on the current loop
2) Force on the stright wire
3) " " " Bented wire

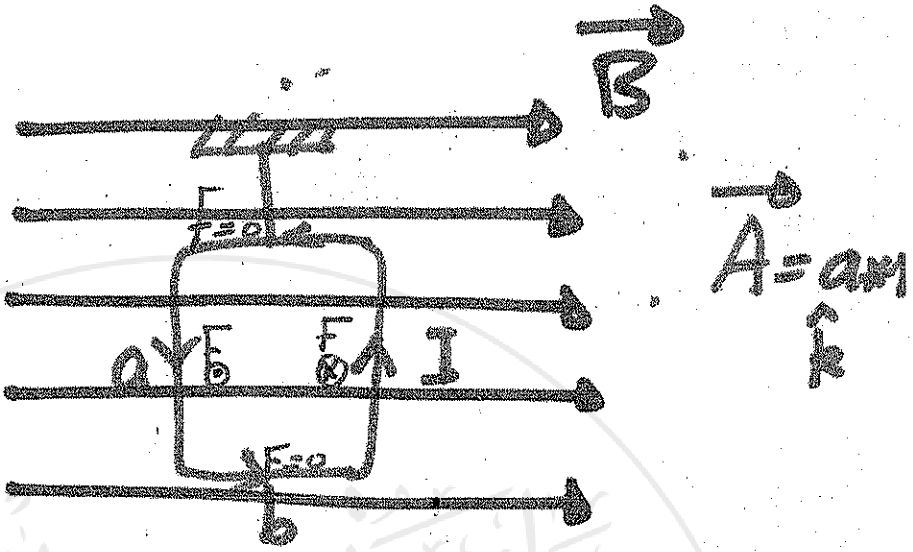



① $F_{net} = 0$

② $\vec{F} = I l B \sin\theta$
$= 2 * 20\times10^{-2} * 4 \sin 90$
$= 1.6$ New. $[2IRB]\ \hat{k}$

③ $F = 2IRB\ (-\hat{k})$
$= 1.6\ (-\hat{k})$ New.

# Torque: ($\tau$)



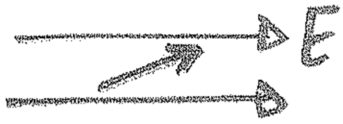

$$\vec{\tau} = NI\vec{A} \times \vec{B}$$

$$\tau = NIAB \sin\theta_{AB}$$

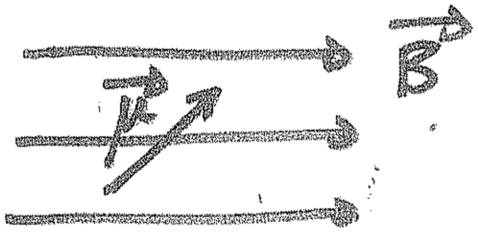
* $\vec{\mu} = NI\vec{A}$ [dipole magnetic moment] $\rightarrow \vec{A}$

$$\Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau = \mu B \sin\theta_{\mu B}$$




$$U = -\vec{P}\cdot\vec{E}$$




U: Potential energy of magnetic dipole moment in $\vec{B}$

$$U = -\vec{\mu}\cdot\vec{B}$$

$$= -\mu B \cos\theta_{\mu B}$$

$$U_{max} = \mu B \quad (\theta = 180^\circ)$$

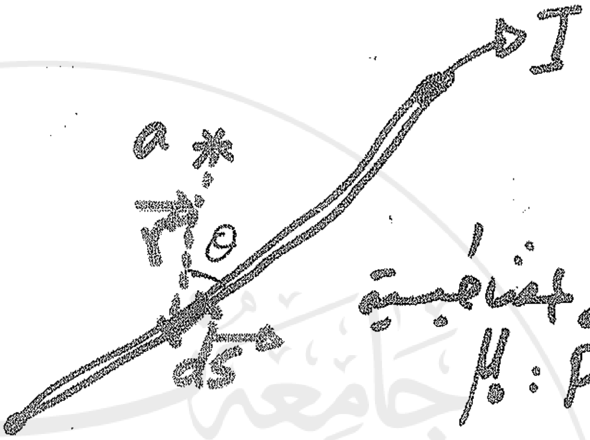
$$U_{min} = -\mu B \quad (\theta = 0)$$

CH:29 انتهى

# CH: ~~30~~ 27 Sources of mag. Field

## 1 Biot - Savart law :-

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$




$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}}$$

النفاذية المغناطيسية

$\mu_0$ : Permeability

$\mu_0 = 4\pi \times 10^{-7}\ T.m/A$

### a) Finite wire

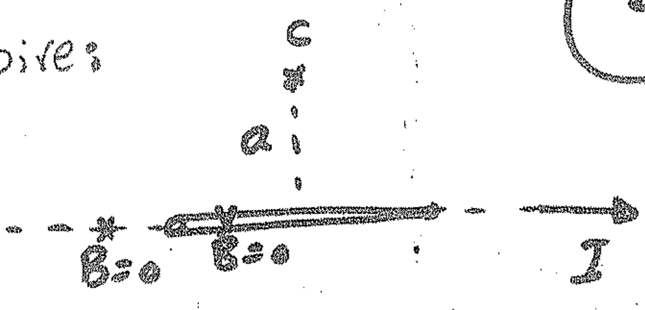

$$B_c = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 - \sin\theta_2]$$

$$\sin\theta_1 = \frac{l_1}{\sqrt{l_1^2 + a^2}}$$

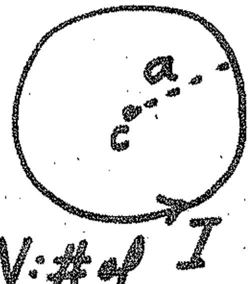
$$\sin\theta_2 = \frac{l_2}{\sqrt{l_2^2 + a^2}}$$

### b) Infinite wire :

$$B_c = \frac{\mu_0 I}{2\pi a}$$



## (c) circular wire




N: # of turns

$$B = \frac{\mu_0 N I}{2a} = \frac{\mu_0 I \theta}{4\pi a}$$

$$N = \frac{\theta}{2\pi}$$


$N = \frac{1}{2}$

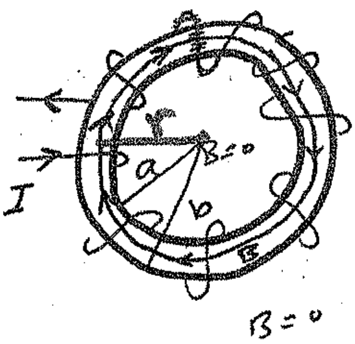

$N = \frac{1}{6}$


$N = \frac{1}{4}$

## (d)


$$B_b = \frac{\mu_0 I a^2 N}{2(a^2+x^2)^{3/2}}$$

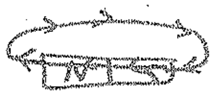
## (e) Toroid:



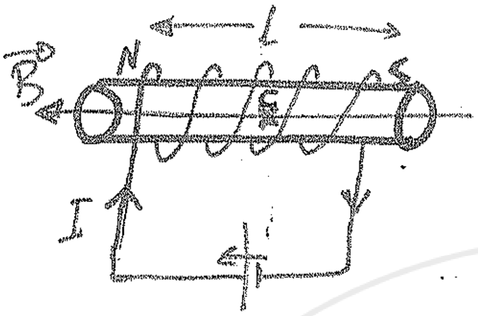

$$B = \frac{\mu_0 N I}{2\pi r}$$

$a < r < b$

(f) <u>Solenoid</u>



③



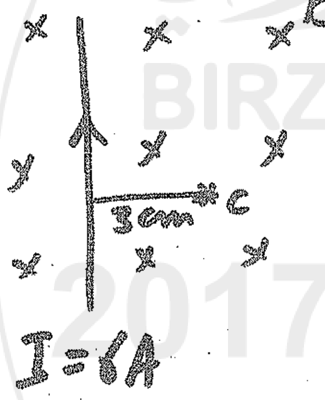

$$B_c = \frac{\mu_0 N I}{L} = \mu_0 n I$$

$\frac{N}{L} = n$ : # of turns per unit length

---

Find the net $\vec{B}$ at $c$ :

<u>Ex</u>: $B = 4\times10^{-5}\,T$




$B_1 \equiv \otimes \; (-\hat{k})$

$B_2 \equiv \otimes \; (-\hat{k})$
stright

١) الاتجاهات
٢) الحسابات
٣) المحصلة

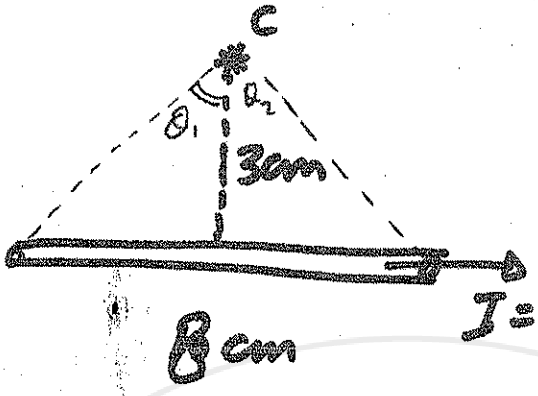
$B_1 = 4\times10^{-5}\,T$

$$B_2 = \frac{\mu_0 I}{2\pi a} = \frac{4\pi\times10^{-7} * 6}{2\pi\, 3\times10^{-2}} = 4\times10^{-5}\,T.$$

$$B_c = B_1 + B_2 = 8\times10^{-5}\,T \otimes (-\hat{k}) .$$

**Ex:**






$\sin\theta_1 = \frac{4cm}{5cm} = 0.8$

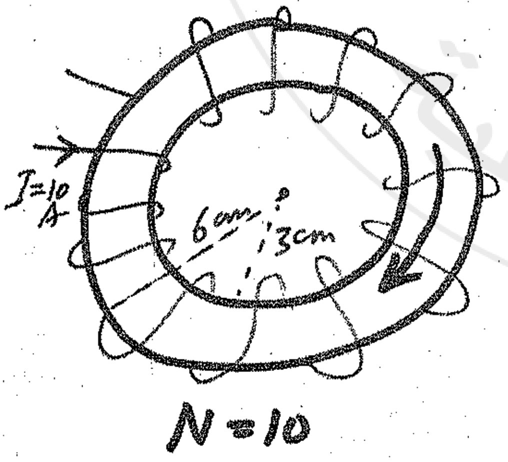
$\sin\theta_2 = \frac{-4cm}{5cm} = -0.8$

$$B_c = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\cancel{4\pi} \times 10^{-7} \times 10}{\cancel{4\pi}\, 3 \times 10^{-2}}(0.8 - (-0.8))$$

$$= \frac{16}{3} \times 10^{-5}\ T$$

**Ex:** Find B at 5cm from the center:

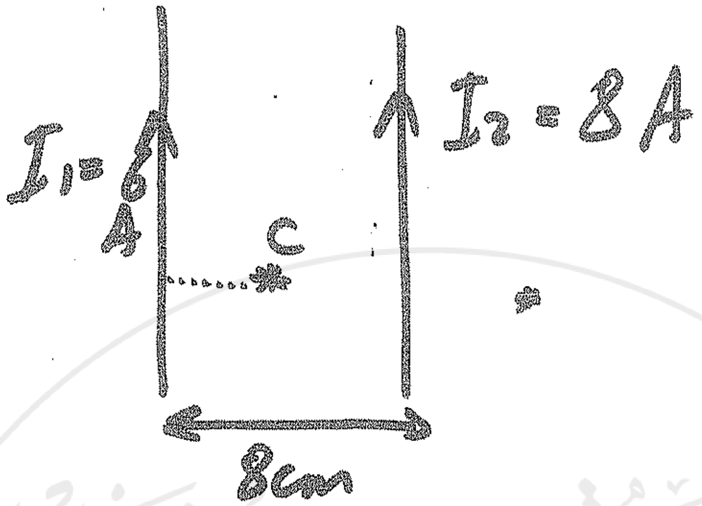



$$B = \frac{\mu_0 N I}{2\pi r}$$

$$= \frac{4\cancel{\pi} \times 10^{-7} \times 10 \times 10}{2\cancel{\pi}\, 5 \times 10^{-2}}$$

$$= 40 \times 10^{-7}\ T$$

Ex: 7




C * $B_1 \otimes$

$B_2 \odot$

$$B_1 = \frac{4\pi \times 10^{-7} * 6}{2\pi \, 4\times 10^{-2}} = 3\times 10^{-5}\ T$$

$$B_2 = \frac{4\pi \times 10^{-7} \times 8}{2\pi * 4\times 10^{-2}} = 4\times 10^{-5}\ T$$

$$B_c = B_2 - B_1 = 1 \times 10^{-5}\ T \odot \ (+z).$$

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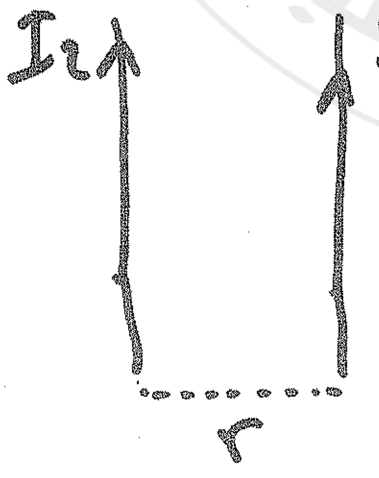
نقطة يتعادل المجالين :

$B_1 = B_2$

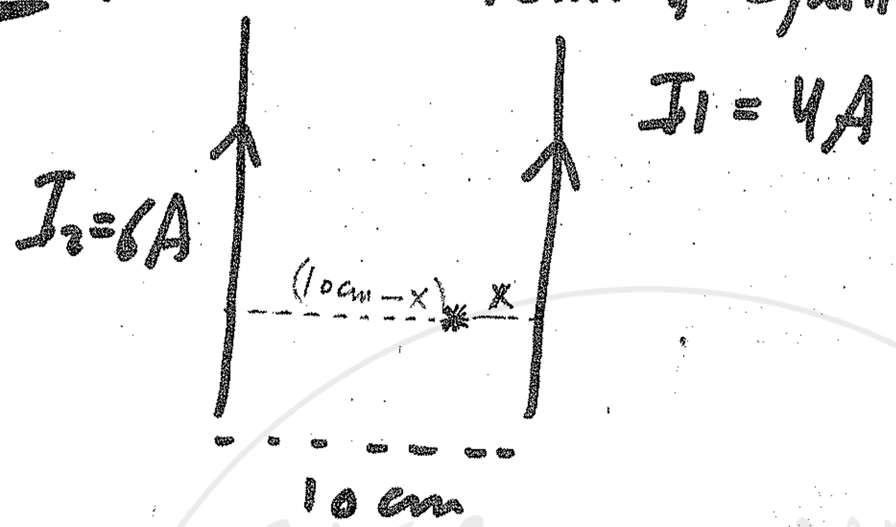
متعاكسان




$\uparrow \ \uparrow$ ⟸ بينهما أقرب للأصغر

$\uparrow \ \downarrow$ ⟸ خارجهما أقرب للأصغر

Ex: Find the Point of equilibrium (where $B_{net} = 0$)




$$B_1 = B_2$$

$$\frac{\cancel{\mu_0} I_1}{\cancel{2\pi} a_1} = \frac{\cancel{\mu_0} I_2}{\cancel{2\pi} a_2}$$

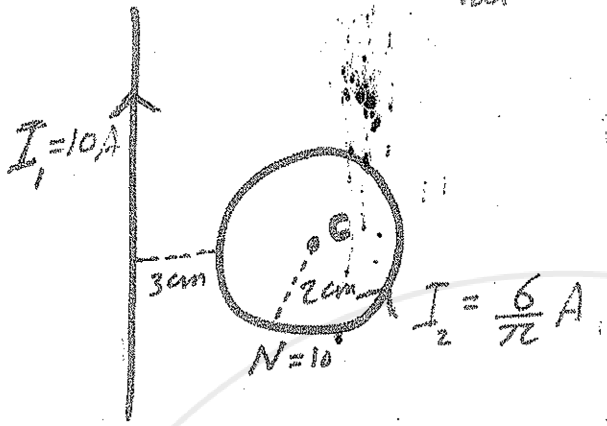
$$\frac{4}{x} = \frac{6}{10 \times 10^{-2} - x}$$

$$\Rightarrow 40 \times 10^{-2} - 4x = 6x$$

$$10x = 40 \times 10^{-2}$$

$$x = 4 \times 10^{-2}\ m = 4\ cm.$$

**Ex:** Find $B_{net}$ at c :-




$*^{c}$ $B_1 \otimes$

$B_2 \odot$

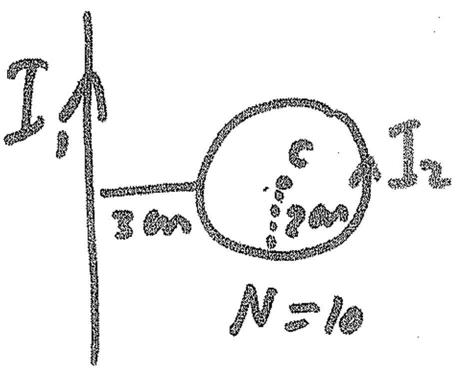
$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{4\pi\times10^{-7}\times10}{2\pi\, 5\times10^{-2}} = 4\times10^{-5}\,T$$

$$B_2 = \frac{\mu_0 I N}{2a} = \frac{4\pi\times10^{-7}\times10\times6}{2\times2\times10^{-2}\pi} = 60\times10^{-5}\,T$$

$$B_{net} = B_2 - B_1 = 56\times10^{-5}\,T \odot (+\hat{k})$$

---

**Ex:** In the previous example, Find $I_2$ if B at c is $1\times10^{-5}\,T\ (-\hat{k})$




$*^{c}$ $B_1 \otimes = 4\times10^{-5}\,T$

$B_c \otimes = 1\times10^{-5}\,T$

$B_2 \odot = 3\times10^{-5}\,T$

$\boxed{\text{if } B_c = 0}$

$B_1 = B_2$

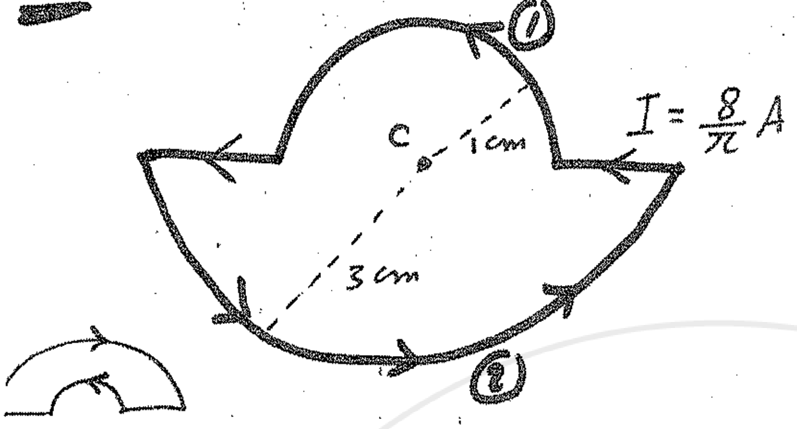
$$\frac{\mu_0 I_1}{2\pi a_1} = \frac{\mu_0 I_2 N}{2 a_2}$$

$$\frac{10}{\pi\, 5\times10^{-2}} = \frac{I_2 \times 10}{2cm}$$

$I_2 = \frac{0.4}{\pi} A$

$$B_2 = \frac{\mu_0 I N}{2a} \Rightarrow 3\times10^{-5} = \frac{4\pi\times10^{-7}\, I_2\, 10}{2\times2\times10^{-2}} \Rightarrow I_2 = \frac{0.3}{\pi} A$$

**Ex:**



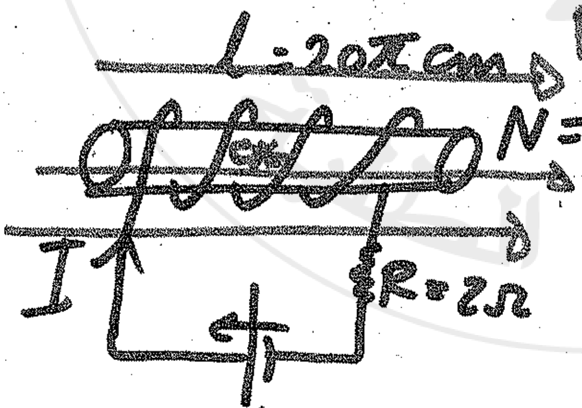

C ×  $B_1 \odot$  $B_2 \odot$

$$B_1 = \frac{\mu_0 N I}{2a} = \frac{4\pi \times 10^{-7} * 1 * 8}{2 \times 1 \times 10^{-2} * 2 * \pi} = 8 * 10^{-5}\,T$$

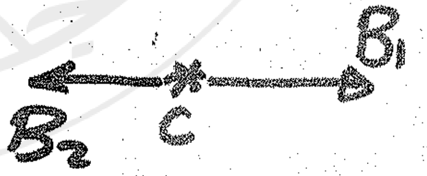
$$B_2 = \frac{4\pi \times 10^{-7} * 1 \times 8}{2 \times 3 \times 10^{-2}\; 2 \times \pi} = \frac{8}{3} \times 10^{-5}\,T$$

$$B_C = B_1 + B_2 = \frac{32}{3} * 10^{-5}\,T \odot (+\hat{k}).$$

**Ex:**




$B_1 = 25 \times 10^{-5}\,T$




$\mathcal{E} = 20V$

$$I = \frac{\mathcal{E}}{R} = 10A$$

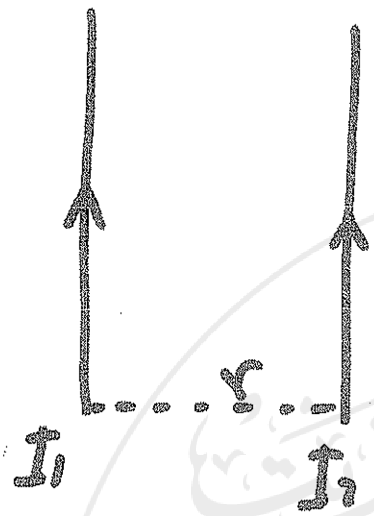
$$B_C = B_1 - B_2$$

$$= 25 \times 10^{-5} - \frac{4\pi \times 10^{-7} \times 10 \times 10}{20\pi \times 10^{-2}}$$

$$= 25 \times 10^{-5} - 20 * 10^{-5}$$

$$= 5 \times 10^{-5} \ldots$$

## Force between the two wires :- (9)




* $\uparrow\uparrow \Rightarrow$ تجاذب

$\downarrow\uparrow \Rightarrow$ تنافر

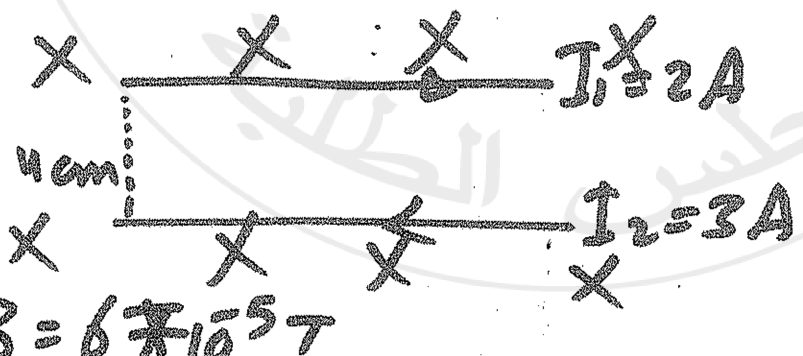
$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

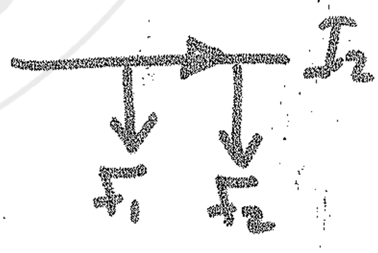
$$\boxed{\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$



---

**Ex** : what is the net Forces on $I_2$ :




$B = 6 \times 10^{-5}\,T$

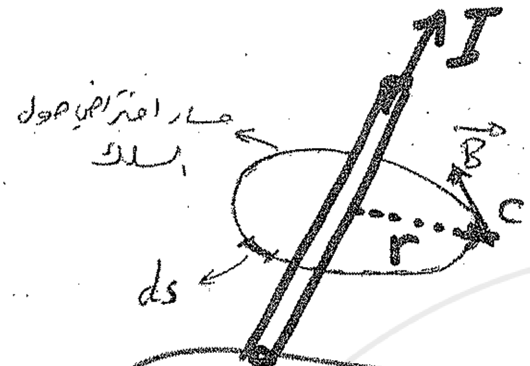
$$\frac{F_1}{L} = \frac{4\pi \times 10^{-7} \times 2 \times 3}{2\pi \times 4 \times 10^{-2}} = 3 \times 10^{-5}\ N/m.$$

$$\frac{F_2}{l} = I_2 B \sin\theta = 3 \times 6 \times 10^{-5} \sin 90 = 18 \times 10^{-5}\ N/m$$

$$\boxed{\frac{F_{net}}{L} = \frac{F_1 + F_2}{L} = 21 \times 10^{-5}\ \frac{N}{m}\ (-\hat{j})}$$

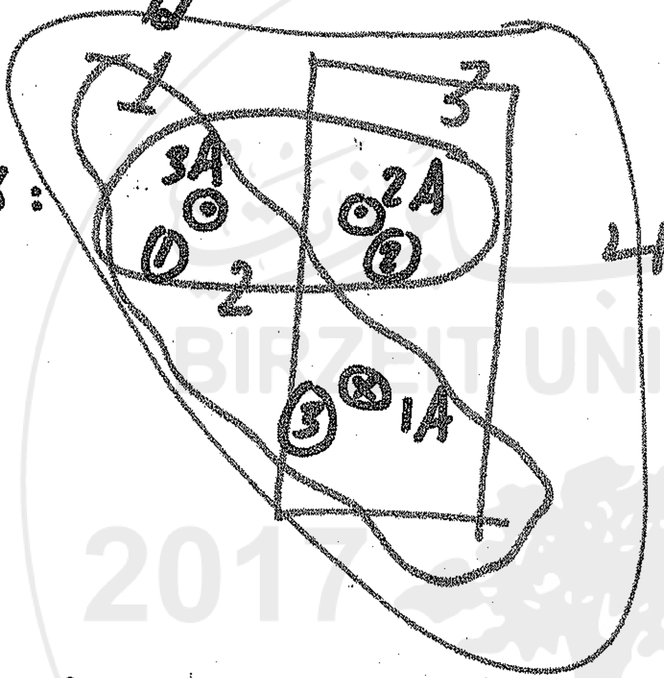
## §3 Ampere's law :-






$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{ins}$$

Ex :




Find the line integral of $\vec{B}\cdot d\vec{s}$ $(\oint \vec{B}\cdot d\vec{s})$ for each line.

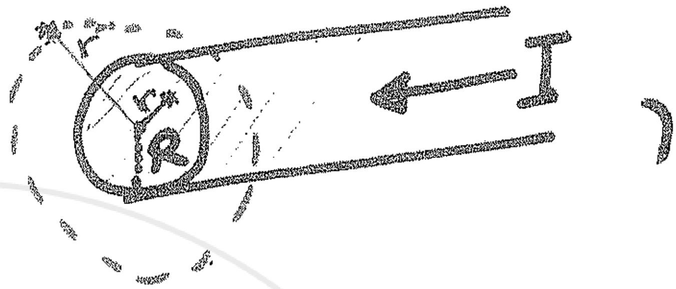
1) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{ins} = 4\pi \times 10^{-7} \times (3-1)$
$= 8\pi \times 10^{-7}$ T.m

2) $\oint \vec{B}\cdot d\vec{s} = \mu_0 (3+2) = 5\mu_0$ T.m

3) $\oint \vec{B}\cdot d\vec{s} = \mu_0 (2-1) = \mu_0$ T.m.

4) $\oint \vec{B}\cdot d\vec{s} = \mu_0 (3+2-1) = 4\mu_0$ T.m.

**Ex**: Find $\vec{B}$ at 1) $r \geq R$
2) $r < R$



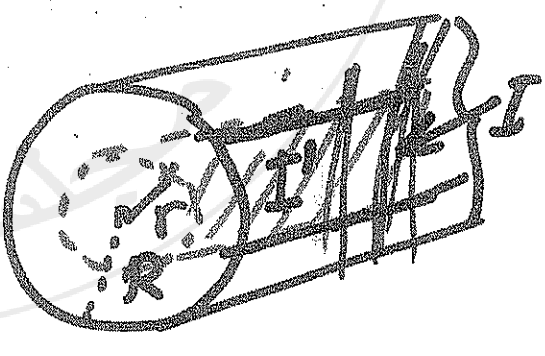
1) $r \geq R$

$$\oint B \cdot d\vec{s} = \mu_0 I_{ins} \qquad \int ds = S_{total} = \text{المحيط}$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B_{out} = \frac{\mu_0 I}{2\pi r}}$$

2) $r < R$



$$B * S = \mu_0 I_{ins}$$

$$B * (2\pi r) = \mu_0 I'$$

$$\frac{I}{I'} = \frac{\cancel{\pi} R^2}{\cancel{\pi} r^2}$$

$$I' = I \frac{r^2}{R^2}$$

$$B = \frac{\mu_0 I r^{\cancel{2}}}{2\pi \cancel{r} R^2}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R^2} r}$$

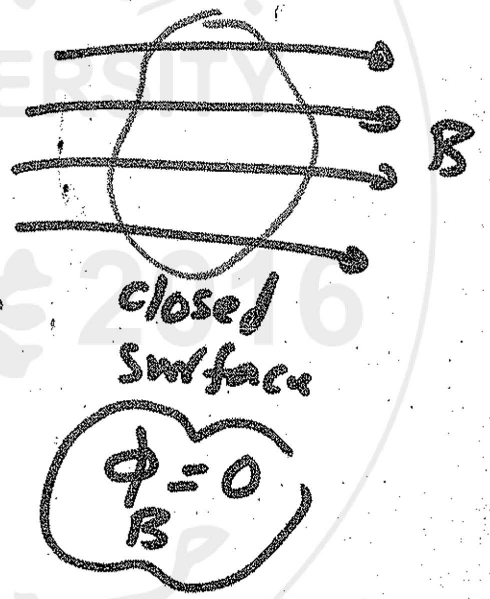
# §5: Gauss's law of magnetism

$\Phi_B$ : magnetic Flux التدفق المغناطيسي

$$\boxed{\Phi_B = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\theta_{BA}}$$

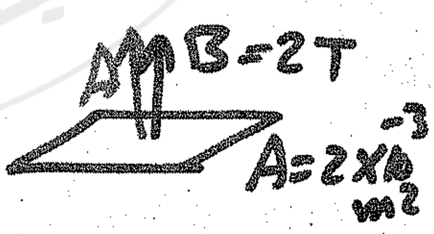
* The net magnetic flux ($\Phi_B$) through any closed surface is always zero

$$\boxed{\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0}$$




Ex:


$\Phi = BA\cos\theta$
$= 2\times 2\times10^{-3}\cos 60$
$= 2\times10^{-3}$ Weber




$\Phi = 2\times 2\times10^{-3}\cos 0$
$\Phi_{max} = 4\times10^{-3}$ Wb.

$\Phi = 0$

Ex: If $\vec{B} = 4\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{A} = 4\hat{i} + 2\hat{j}$

Find Flux (magnetic) and $\theta_{BA}$

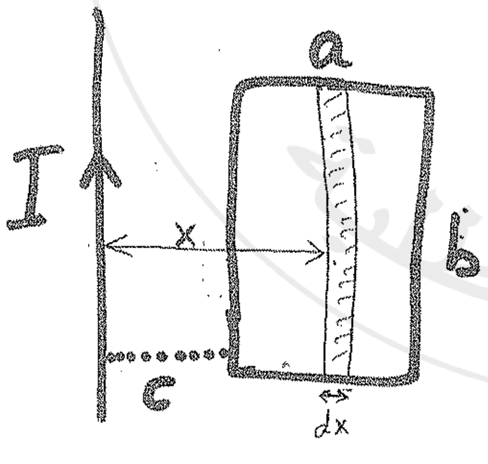
$$\Phi = \vec{B} \cdot \vec{A} = 16 + 6 = 22 \text{ Wb}$$

$$\Phi = BA\cos\theta$$

$$22 = \sqrt{4^2+3^2+5^2} \cdot \sqrt{4^2+2^2} \cos\theta_{BA}$$

---

Ex:




$x: c \longrightarrow c+a$

$dA = b * dx$

$|\vec{B}| = \frac{\mu_0 I}{2\pi x}$

$$\Phi = \int \frac{\mu_0 I}{2\pi x} \cdot b\,dx$$

$$= \frac{\mu_0 I}{2\pi} b \int \frac{dx}{x}$$

$$= \frac{\mu_0 I b}{2\pi} \ln x \Big]_c^{c+a}$$

CH:30 انتهى

$$\Phi = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{c+a}{c}\right)$$

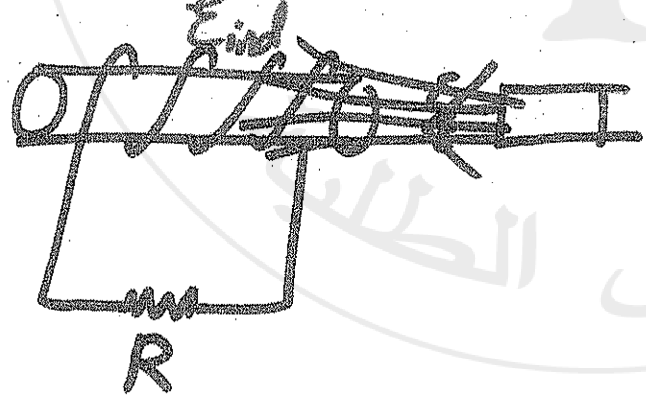
# CH: ~~31~~ 28 Faraday's law



$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\theta_{BA}$$

$$\Delta\Phi = \begin{cases} \Delta B\, A\cos\theta \longrightarrow (\Delta B = B_2 - B_1) \\ B\,\Delta A\cos\theta \longrightarrow (\Delta A = A_2 - A_1) \\ B A\,\Delta\cos\theta \longrightarrow (\Delta\cos\theta = \cos\theta_2 - \cos\theta_1) \\ \Phi_2 - \Phi_1 = B_2 A_2\cos\theta_2 - B_1 A_1\cos\theta_1 \end{cases}$$

$$d\Phi = d(BA\cos\theta) = A\cos\theta\, dB$$




$$\mathcal{E}_{ind} = -N\frac{d\Phi}{dt}$$

$$\mathcal{E}_{ind} = -N\frac{\Delta\Phi}{\Delta t} \longrightarrow I_{ind} = \frac{|\mathcal{E}_{ind}|}{R}$$

* magnitude of $\mathcal{E}_{ind}$ is $|\mathcal{E}_{ind}|$

Ex: A plane of dimentions $10cm \times 6cm$,
and a Uniform mag field $B = 4T$ directed
out of page Perp to the plane. Find the

1) electromotive force (induced).
2) induced Curret ($R = 2\Omega$).

if the B is dropped to zero through
0.2 sec.



$B = 4T \downarrow 0T$ , $A = 60 \times 10^{-4}$ $\theta = 0$ , $\Delta t = 0.2 sec.$

1) $\Delta \phi = \Delta B \; A \cos\theta$
$= (0 - 4) \, 60 \times 10^{-4} \cos 0$
$= -240 \times 10^{-4}$ Wb.

$$\mathcal{E}_{ind} = -N \frac{\Delta \phi}{\Delta t} = -1 \times \frac{-240 \times 10^{-4}}{0.2} = +120 \times 10^{-3} \, wb/s$$

2) $$I_{ind} = \frac{|\mathcal{E}|}{R} = \frac{120 \times 10^{-3}}{2} = 60 \times 10^{-3} A.$$

**Ex**: If $B = B_{max} e^{-\alpha t}$, Find the (3)

induced $\mathcal{E}$ through a surface of Area

**A** parallel to the field. ($\theta = 0$)

**Sol**

$$\phi = BA\cos\theta$$

$$= AB_{max} e^{-\alpha t}$$

$$\frac{d\phi}{dt} = AB_{max}(-\alpha e^{-\alpha t})$$

$$= (-\alpha AB_{max} e^{-\alpha t})$$

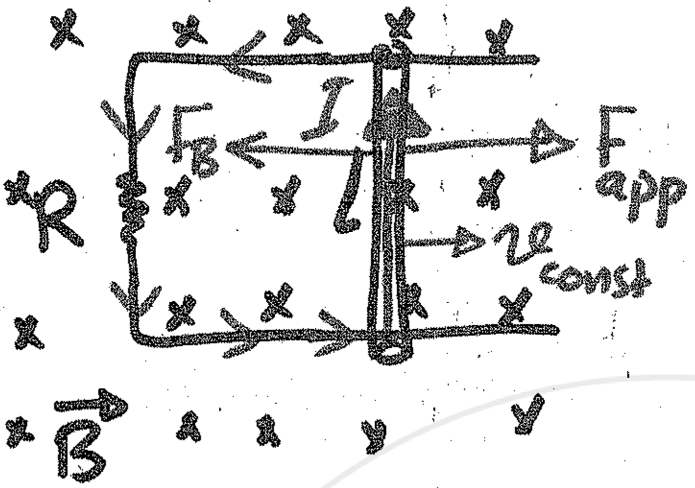
$$\mathcal{E}_{ind\,(ins)} = -N\frac{d\phi}{dt} = \alpha AB_{max} e^{-\alpha t}$$

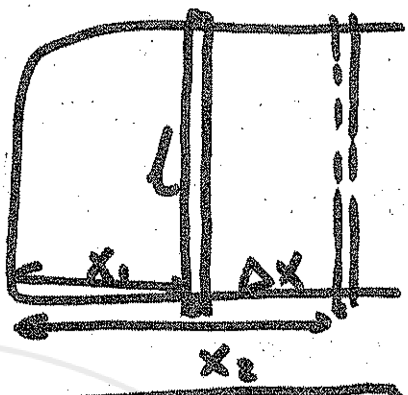
$$* \; \mathcal{E}_{av} = -\frac{\Delta\phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

from $t = 0 \to 2$

$$\phi_1 = AB_1$$

$$\phi_2 = AB_2$$




$$\boxed{\Delta A = l \cdot \Delta x}$$

$$\Delta \Phi = B \, \Delta A \cos 0$$

$$\mathcal{E}_{in} = -\frac{\Delta \Phi}{\Delta t}$$

$$\mathcal{E}_{ind} = -l v B \qquad v \rightarrow \frac{\Delta x}{\Delta t}$$

$$\frac{|\mathcal{E}|}{R} = I$$

$$F_{app} = F_B = I l B \sin_{lB}$$

$$\text{Power} = F_{app} v = I l B v$$

$$\boxed{\text{Power} = \frac{B^2 l^2 v^2}{R} = \mathcal{E}^2 / R}$$

---

If $V_{ini} \neq 0$ ( V is not constant)

$$\Rightarrow v = v_{initial} \, e^{-t/\tau} \qquad \left( \tau = \frac{mR}{B^2 l^2} \right)$$

Ex In the figure, Find: ⑤

1) $\Delta x$

2) $\Delta t$

3) $\Delta A$

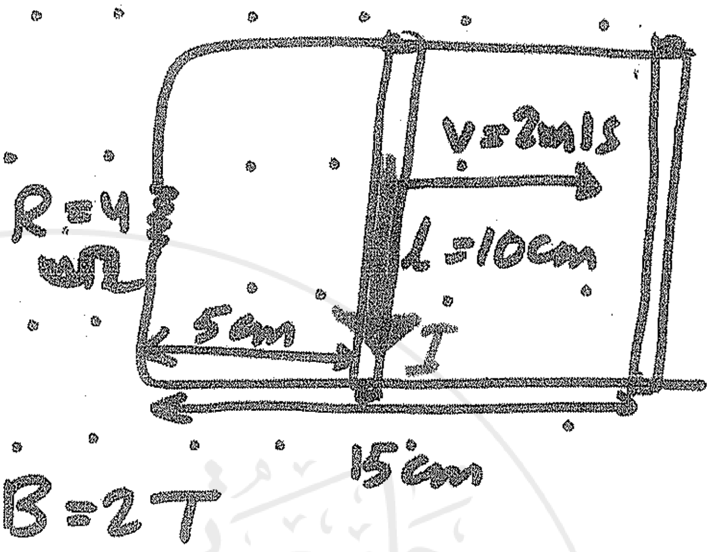
4) $\Delta \phi$

5) $\mathcal{E}$

6) $I$

7) $F_{app}$ , $F_B$

8) Power.






Sol

1) $\Delta x = 15 - 5\,cm$
$= 10 \times 10^{-2} m$

2) $V = \frac{\Delta x}{\Delta t}$
$\Rightarrow \Delta t = \frac{10 \times 10^{-2}}{2}$
$= 5 \times 10^{-2} sec.$

3) $\Delta A = l * \Delta x$
$= 10 \times 10^{-2} * 10 \times 10^{-2}$
$= 100 * 10^{-4} m^2$.

4) $\Delta \phi = B\, \Delta A \cos\theta = 2 * 1 \times 10^{-2} \cos 0 = 2 \times 10^{-2} Wb$

5) $\mathcal{E}_{ind} = -\frac{\Delta \phi}{\Delta t} = -\frac{2 \times 10^{-2}}{5 \times 10^{-2}} = -0.4$ Volt.

6) $I_{in} = \frac{|\mathcal{E}|}{R} = \frac{0.4}{4} = 0.1 A \; [\; -\hat{j} \;]$

Fapp $+i$

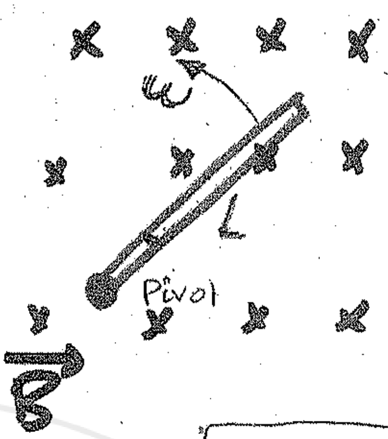
7) $\underline{F_{app}} = F_B = I l B = 0.1 * 10 \times 10^{-1} * 2 = 2 \times 10^{-1} N$ $-i$ $F_B$

8) Power $= \mathcal{E}^2/R = 0.16/4 = 4 * 10^{-2}$ watt.

⑥

$$\boxed{\mathcal{E} = \frac{1}{2} B \omega l^2}$$

$$I_{in} = \frac{|\mathcal{E}|}{R} \qquad T = \frac{2\pi}{\omega}$$

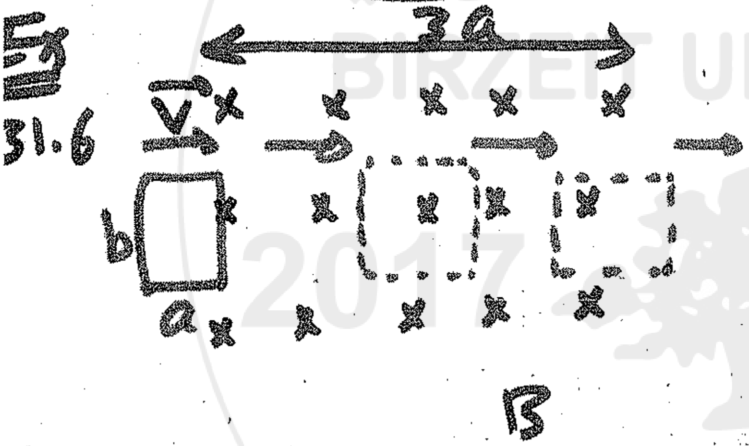



$$\boxed{\omega = \frac{v}{r}}$$

ω : angular Velocity (rad/sec)

T : Period

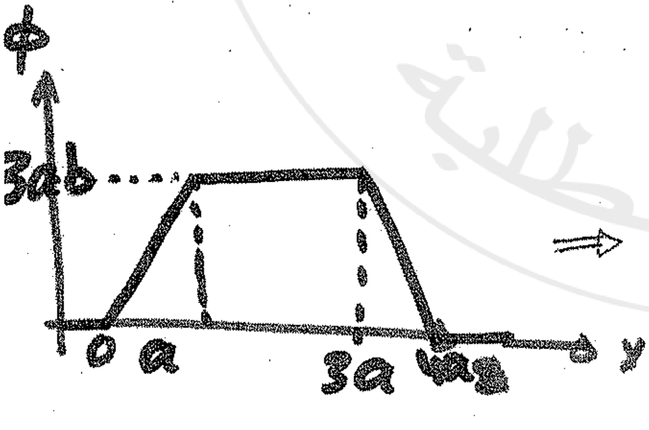
Ex 31.6



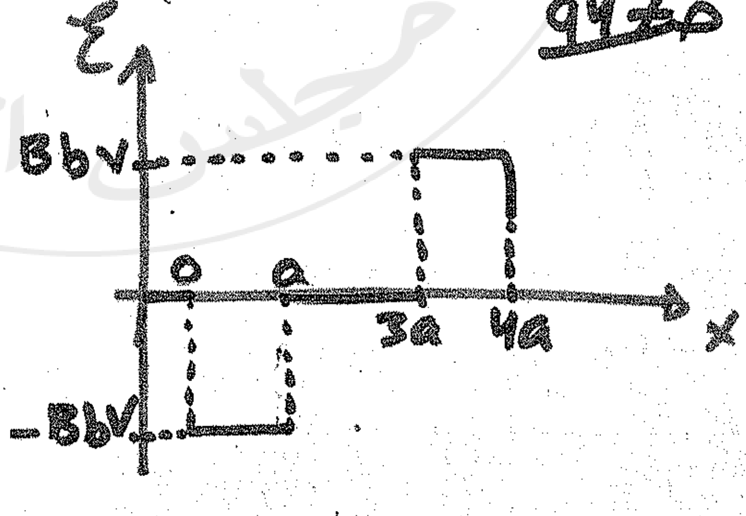

$$\frac{|\mathcal{E}|}{R}$$

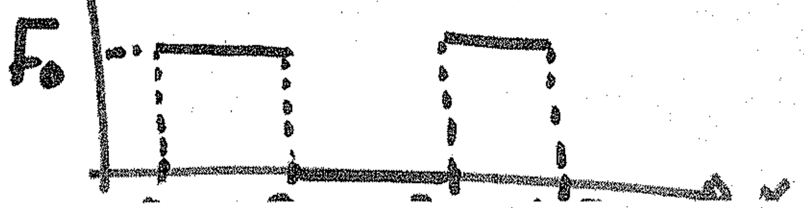
$$F = I l \times B$$

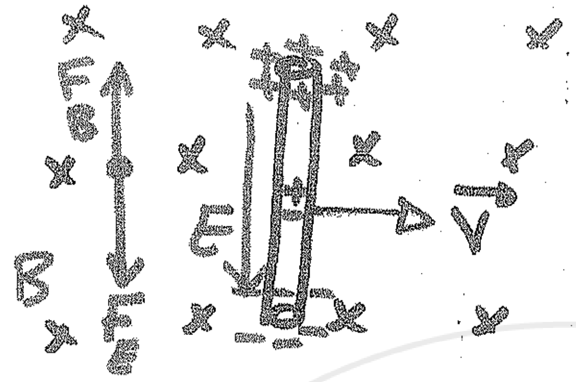
$$F_0 = \frac{v l^2 B^2}{R}$$




94 ص





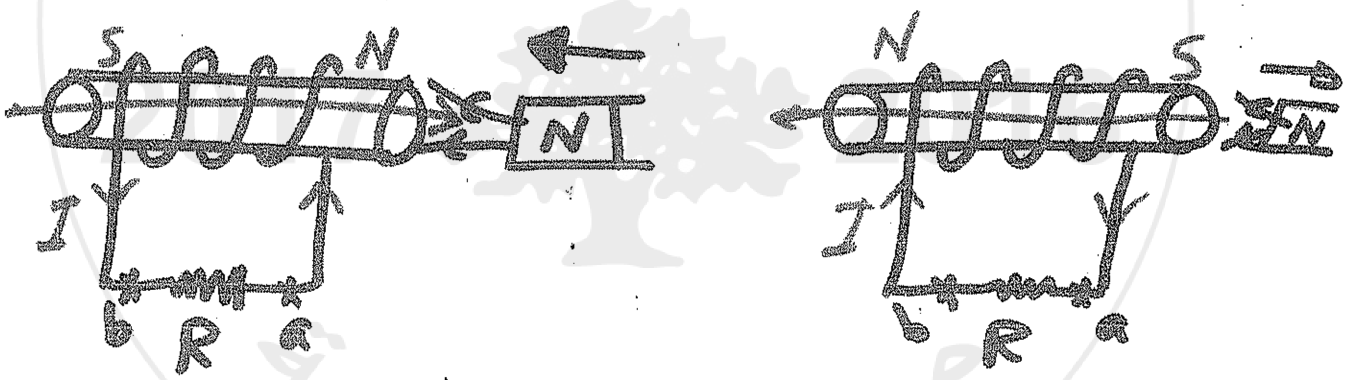

$$F_B = q v B \sin\theta$$

$$F_E = F_B$$

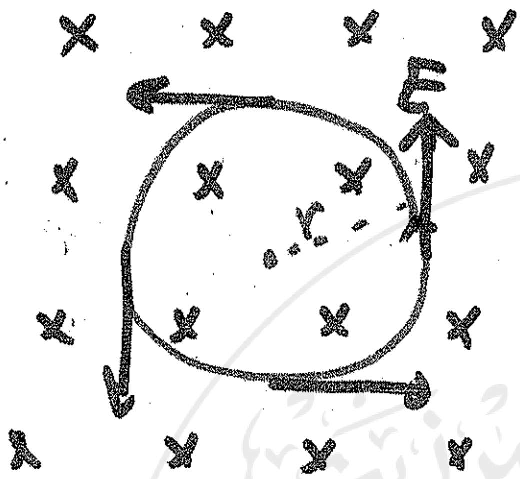
$$qE = qvB$$

$$v = \frac{E}{B}$$

## Lenz law :-




# §4: Induced emf and Electric fields




B: depent on time

$B = B(t)$

$$\frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} \qquad A \begin{cases} A = \pi r^2 \\ a \times b \end{cases}$$

$$\mathcal{E} = -N\frac{d\phi}{dt} = -NA\frac{dB}{dt}$$

$$\cancel{q}\,\mathcal{E} = \cancel{q}\,E(2\pi r) \qquad \frac{\Delta V = E\cdot l}{l}$$

$$E = \frac{\mathcal{E}}{2\pi r} = -\frac{1}{2\cancel{\pi}\cancel{r}} N\cancel{\pi} r^{\cancel{2}}\frac{dB}{dt}$$

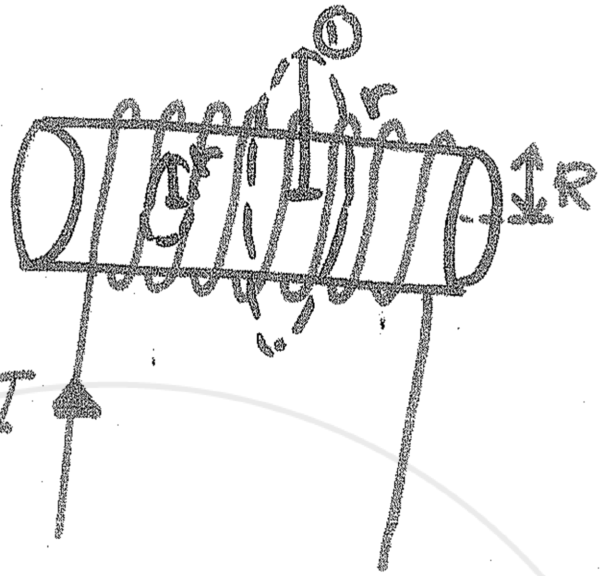
$$E_{in} = -\frac{N}{2}\, r\,\frac{dB}{dt}$$

$$\oint E\cdot d\vec{s} = -N\frac{d\phi}{dt}$$

$$E\cdot \Delta L = \mathcal{E}$$

Ex 31.7
948

(9)




$n = \frac{N}{L}$

$I = I_{max} \cos\omega t$

1) r > R

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(\mu_0 n I \pi R^2)$$

$$= -\mu_0 n \pi R^2 \frac{dI}{dt}$$

$$= -\mu_0 n \pi R^2 (-I_{max}\, \omega \sin\omega t)$$

$$\mathcal{E} = \mu_0 n \pi \omega R^2 I_{max} \sin\omega t$$

$$E(\Delta L) = \mathcal{E}$$

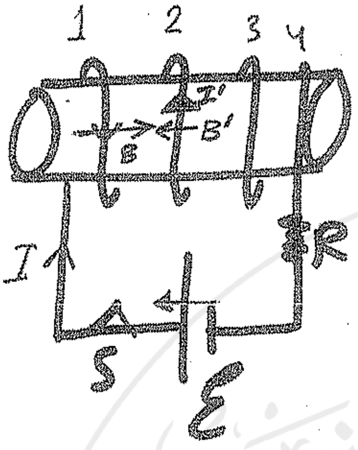
$$E(2\cancel{\pi} r) = \mu_0 n \cancel{\pi} \omega R^2 I_{max} \sin\omega t$$

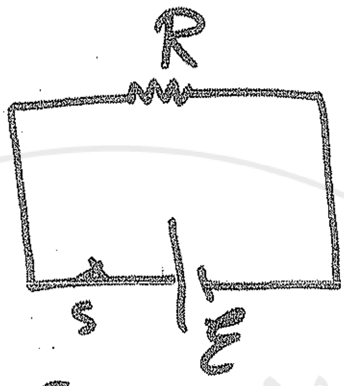
$$\boxed{E_{out\,(r>R)} = \frac{\mu_0 n \omega R^2 I_{max}}{2r} \sin\omega t}$$

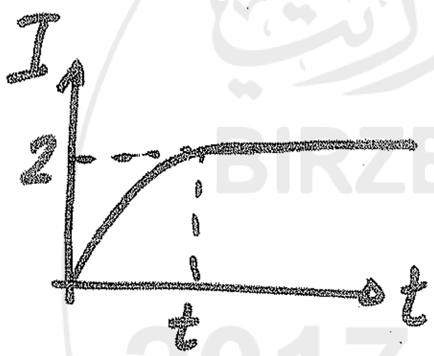
$$\boxed{E_{ins} = \frac{\mu_0 n \omega I_{max}}{2} r \sin\omega t}$$

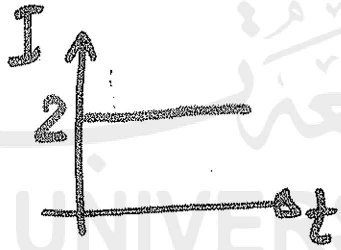
CH:31
أشرف

# CH: ~~32~~ 29 Self Induction and Inductance

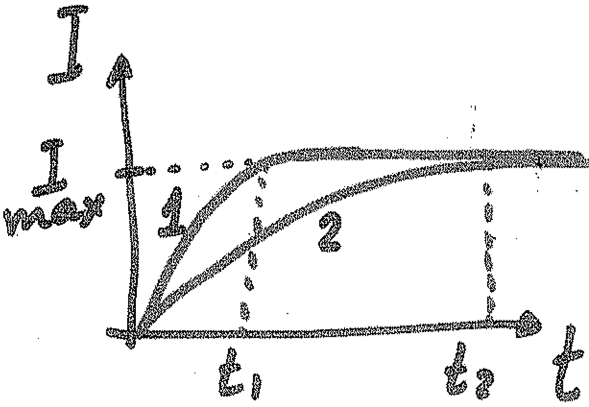







Inductance
$\Downarrow$
L

$L_2 > L_1$

## Self Induction:

②

$$\mathcal{E}'_L = -N\frac{d\phi}{dt} = -N\frac{\Delta\phi}{dt}$$ (general)

$\phi = BA\cos\theta$

$d\phi = d(BA\cos\theta)$

$\Delta\phi = \Delta(BA\cos\theta)$

$$\mathcal{E}'_L = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t}$$

$$i' = \frac{|\mathcal{E}'|}{R}$$

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$$L_{general} = \frac{N\Delta\phi}{\Delta i} = \frac{N\phi_B}{i}$$

$$L_{solenoid} = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 l A = \mu_0 n^2 Vol$$

$L \longrightarrow$ Henry (H)

$C = \frac{q}{V}$

$R = \frac{V}{i}$

$L = \frac{N\phi}{i}$

$N = nl$

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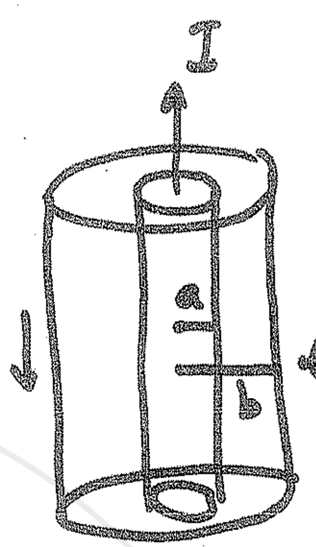
$$U_B = \frac{1}{2}LI^2$$ (magnetic energy stored in the magnetic field inside the inductor)

$$u_B = \frac{U}{Vol} = \frac{B^2}{2\mu_0}$$ energy density or " per unit

$u_E = \frac{1}{2}\epsilon_0 E^2$

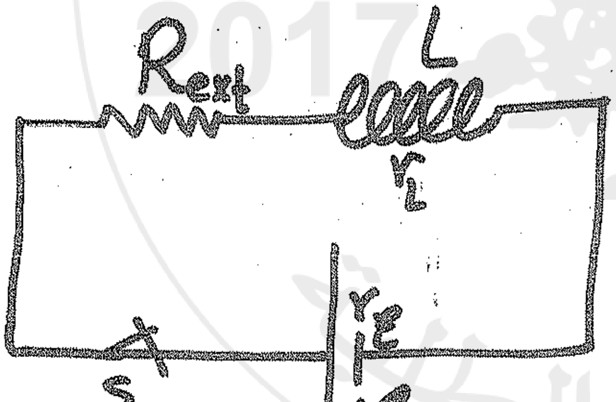
$$\boxed{L_{\text{coaxial cable}} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)}$$




$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$



RL - circuit :




$$R = R_{ext} + r_L + r_\mathcal{E}$$

$$\mathcal{E} = iR + L\frac{di}{dt}$$

تشغيل الدارة

| | $t=0$ | $t=\infty$ |
|---|---|---|
| $i$ | $0$ | $i_{max}$ |
| $\frac{di}{dt}$ | $\left(\frac{di}{dt}\right)_{max}$ | $0$ |

$$i_{max} = \frac{\mathcal{E}}{R}$$

time constant $\tau$

$$\left(\frac{di}{dt}\right)_{max} = \frac{\mathcal{E}}{L}$$

$$\tau = \frac{L}{R}$$

| $i$ | $di/dt$ |
|---|---|
| 1) $i$ | 1) $di/dt$ |
| 2) $V_R = iR$ | 2) $V_L = L\frac{di}{dt} + i r_L$ |
| 3) $V_\mathcal{E} = \mathcal{E} - i r_\mathcal{E}$ | 3) $\mathcal{E}' = -L\,di/dt$ |
| 4) $P_R = i^2 R$ | 4) $Power_{(L)} = i L \frac{di}{dt}$ |
| 5) $P_\mathcal{E} = i\mathcal{E}$ | |
| 6) $U_L = \frac{1}{2} L i^2$ | |

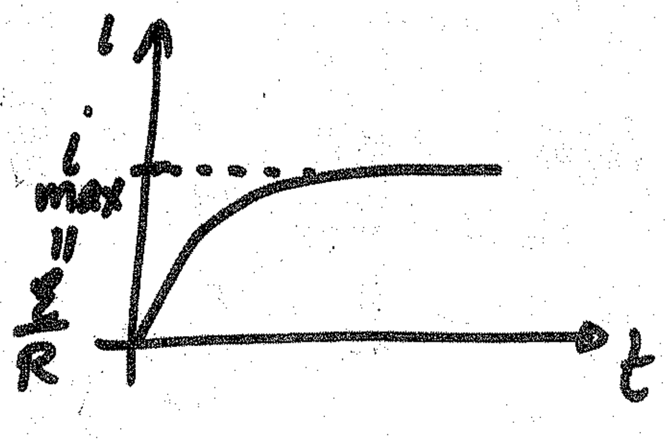
$$\mathcal{E} = iR + L\frac{di}{dt}$$

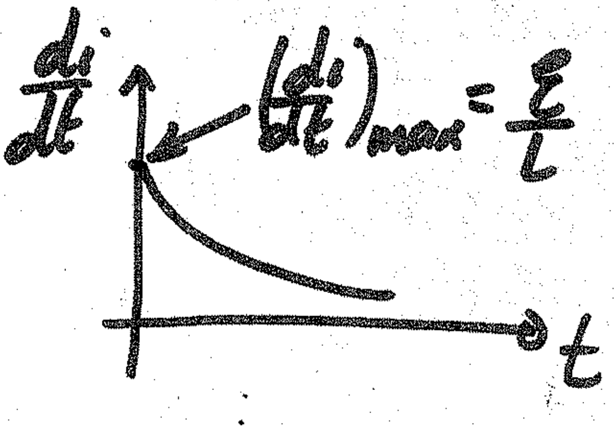
$$i = \frac{1}{2} i_{max}$$

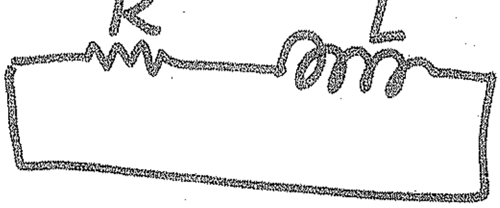


$$i = i_{max}\left(1 - e^{-Rt/L}\right) \qquad \tau = \frac{L}{R}$$

$$\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-Rt/L}$$



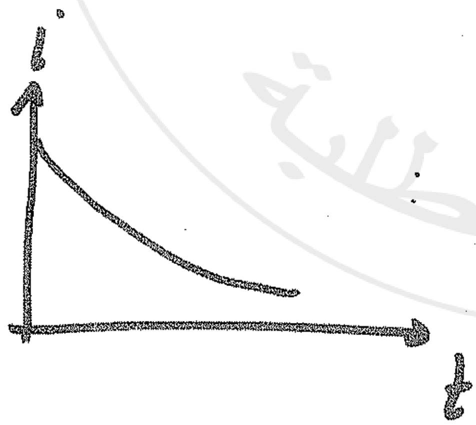


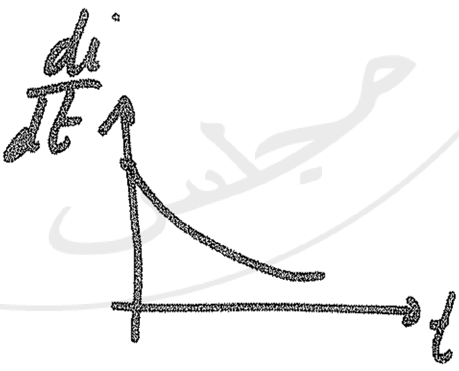

$$0 = iR + L\frac{di}{dt}$$

$$i_{max} = \frac{\varepsilon}{R}, \quad \left(\frac{di}{dt}\right)_{max} = \frac{\varepsilon}{L}, \quad \tau = \frac{L}{R}$$

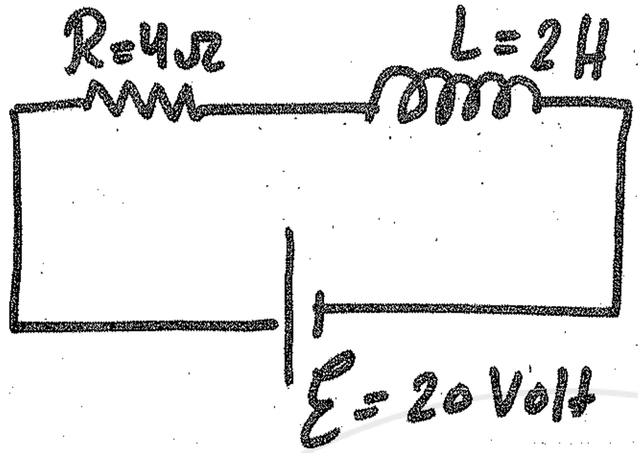
$$i = i_{max}\, e^{-t/\tau}$$

$$\left(\frac{di}{dt}\right) = -\left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$$



Ex:




Find: 1) $i_{max}$ , $\left(\frac{di}{dt}\right)_{max}$ , $\tau$

2) if $i = \frac{1}{5} i_{max}$ , Find:

a) $\frac{di}{dt}$  b) Voltage accross the inductor

c) induced emf ($\mathcal{E}'$)  d) Power in L.

3) if $\frac{di}{dt}$ is equal to 40% of its max. value, what is:

a) current ($i$)  b) Voltage accross (R)

c) Power in R  d) " " ($\mathcal{E}$)

e) " " $\mathcal{E}$  f) energy in the inductor

4) at $t = 2$-sec, find:

a) current  b) change rate of current $\left(\frac{di}{dt}\right)$

5) find Current (i) and $\frac{di}{dt}$ after 3-time constant

6) find time needed to reach to $\frac{1}{2}$ max. Current

7) " " " " " $\frac{1}{10}$ " $\frac{di}{dt}$.

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Sol: $R = 4\Omega$ , $L = 2H$ , $\mathcal{E} = 20$ Volt

① $\tau = \frac{L}{R} = \frac{2}{4} = 0.5$ sec

$$i_{max} = \frac{\mathcal{E}}{R} = \frac{20}{4} = 5A$$

$$\left(\frac{di}{dt}\right)_{max} = \frac{\mathcal{E}}{L} = \frac{20}{2} = 10 A/s$$

2) $\boxed{i = \frac{1}{5} i_{max} = \frac{1}{5} * 5 = 1A}$ $\left(\frac{1}{5} = 20\%\right)$ ⑧

a) $\mathcal{E} = iR + L\frac{di}{dt}$

$20 = 1*4 + 2\frac{di}{dt}$

$\boxed{\frac{di}{dt} = 8 A/s}$

b) $V_L = L\frac{di}{dt} + \cancel{iR}$

$= 2*8$

$= 16$ Volt

c) $\mathcal{E}' = -L\frac{di}{dt} = -16V$

d) $P_L = i L\frac{di}{dt}$

$= 1*2*8$

$= 16$ watt

3) $\frac{di}{dt} = \frac{40}{100}\left(\frac{di}{dt}\right)_{max} = \frac{40}{100} * 10 = 4 A/s$.

a) $\mathcal{E} = iR + L\frac{di}{dt}$

$20 = i*4 + 2*4$

$\boxed{i = 3A}$

b) $V_R = iR$

$= 3*4$

$= 12$ Volt

c) $P_R = i^2 R$

$= 9*4 = 36$ Volt

(d) $V_{\mathcal{E}} = \mathcal{E} - \cancel{ix}$

$= 20$ Volt

---

(e) $P_{\mathcal{E}} = i\mathcal{E} = 3 * 20 = 60$ watt

---

(f) $U_L = \frac{1}{2} L i^2$

$= \frac{1}{2} * 2 * (3)^2 = 9$ J.

---

(g) $\underline{t = 2}$ $\quad \tau = 0.5$ sec

$i = i_{max}\left(1 - e^{-t/\tau}\right)$

$= 5\left(1 - e^{-2/0.5}\right) = 4.9$ A

---

$\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$

$= 10 \; e^{-2/0.5} = 0.18$ A/s

(5) $t = 3\tau$ 



$$i = 5(1 - e^{-3\cancel{\tau}/\cancel{\tau}}) = 4.75\,A.$$

$$\frac{di}{dt} = 10\, e^{-3\cancel{\tau}/\cancel{\tau}} = 0.5\ A/s.$$

(6) $i = i_{max}(1 - e^{-t/\tau})$

$$\frac{1}{2}\cancel{i_{max}} = \cancel{i_{max}}(1 - e^{-t/\tau})$$

$$\frac{1}{2} = 1 - e^{-t/0.5}$$

$$\cancel{-}\frac{1}{2} = \cancel{-}e^{-t/0.5} \Rightarrow \ln \Rightarrow \frac{-t}{0.5} = \ln 0.5 = -0.693$$

$$\boxed{t = 0.35\ sec}$$

(7) $\frac{di}{dt} = \left(\frac{di}{dt}\right)_{max} e^{-t/\tau}$

$$0.1\cancel{\left(\frac{di}{dt}\right)_{max}} = \cancel{\left(\frac{di}{dt}\right)_{max}} e^{-t/\tau} \Rightarrow 0.1 = e^{-t/\tau} \Rightarrow \ln$$

$$\ln 0.1 = \frac{-t}{0.5} \Rightarrow \boxed{t = 1.15\ sec}$$

ch:32 انتهى